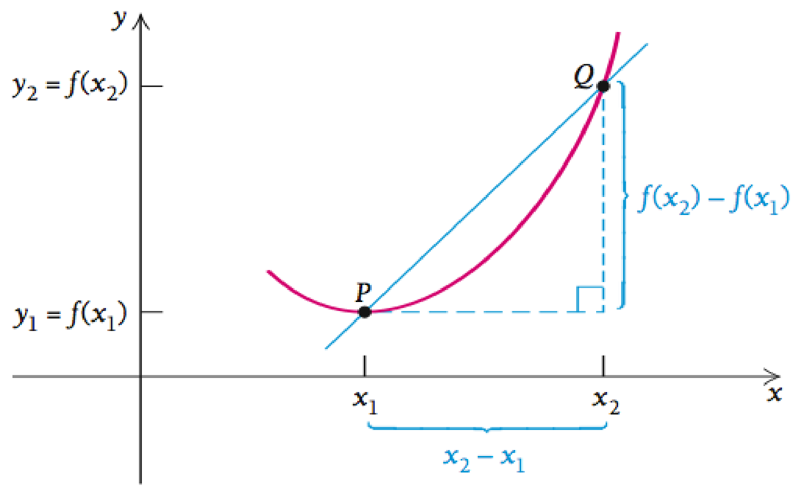
Average Rate of Change

The **average rate of change of *y* with respect to *x***, as x changes from x_1 to x_2, is the ratio of the change in output to the change in input:

$$\frac{y_2 - y_1}{x_2 - x_1}, \qquad \text{where } x_2 \neq x_1.$$

If we look at a graph of the function, we see that $\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$, which is both the average rate of change *and* the slope of the line from $P(x_1, y_1)$ to $Q(x_2, y_2)$. The line through P and Q, $\overline{PQ}$, is called a **secant line**.



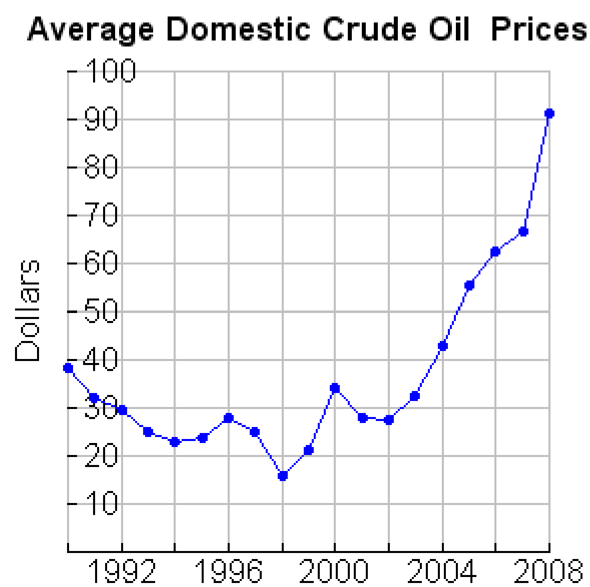
Example:

The graph shows the average cost of a barrel of crude oil from 1990 to 2008. Prices are adjusted for Inflation to November 2008 prices using the Consumer Price Index (CPI-U) as presented by the Bureau of Labor Statistics.

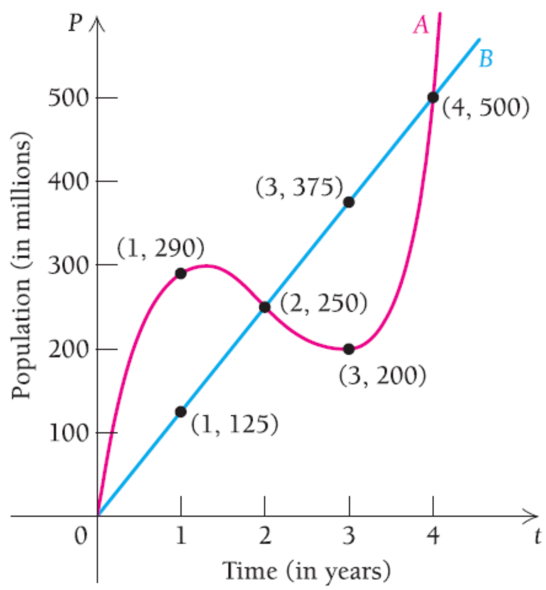
Find the approximate average change in price over the given time periods.

1a. 1992 to 1994 b. 1998 to 2004

2a. 2002 to 2008 b. 1990 to 19998



Population growth. The two curves below describe the numbers of people in two countries at time t, in years.



a) Find the average rate of change of each population with respect to time t as t changes from 0 to 4. This is often called the **average growth rate.**

b) If the calculation in part (a) were the only one made, would we detect the fact that the populations were growing differently? Explain.

c) Find the average rates of change of each population as t changes from 0 to 1; from 1 to 2; from 2 to 3; from 3 to 4.

d) For which population does the statement "the population grew consistently at a rate of 125 million per year" convey accurate information? Why?

Given the function: $f(x) = \frac{1}{x}$

a) Find a simplified form of the difference quotient, $\frac{f(x+h) - f(x)}{h}$

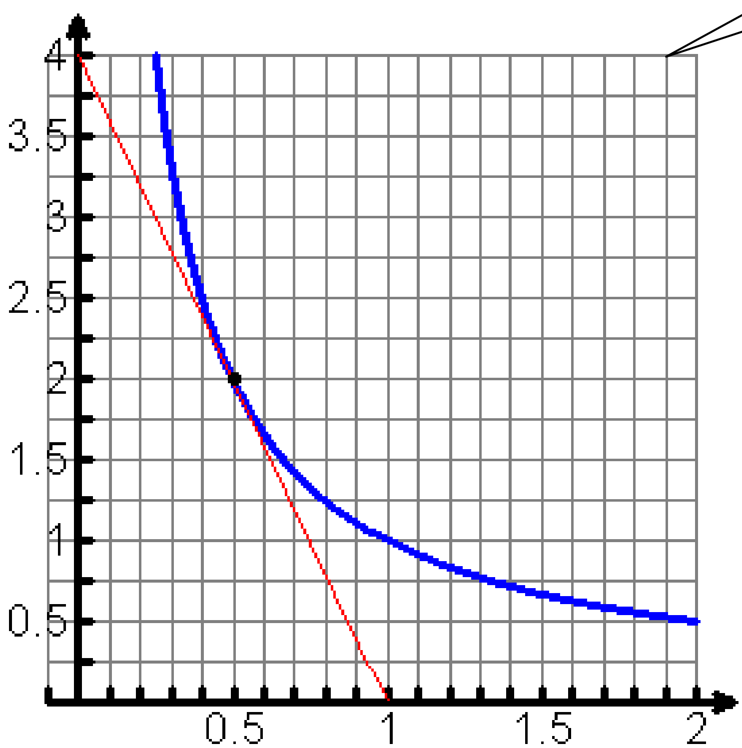
b) Complete the table below.

c) Sketch each secant line on the graph below.

x	h	$\frac{f(x+h)-f(x)}{h}$
0.5	1.5	-1
0.5	1	
0.5	0.5	
0.5	0.25	
0.5	0.1	
0.5	0.05	
0.5	0.02	
0.5	0.01	
0.5	0.001	
0.5	0.0001	

$$\frac{f(0.5+1.5)-f(0.5)}{1.5} = \frac{f(2)-f(0.5)}{1.5} = \frac{0.5-2}{1.5} = -1$$

No crooked lines - Use a straight edge.



The slopes of the

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approach the slope of the

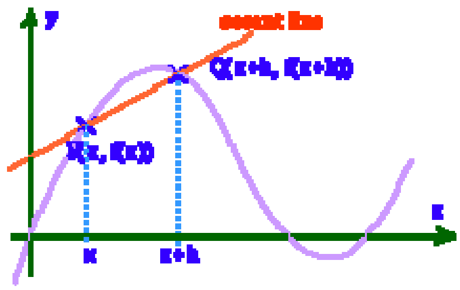
tangent line.

The Derivative

http://www.zweigmedia.com/RealWorld/tutorials/frames2_3.html

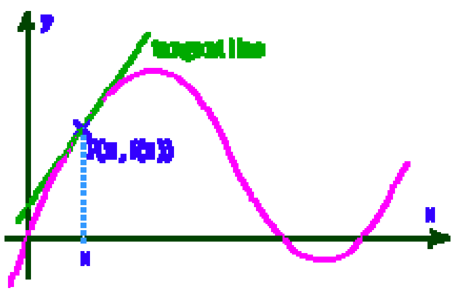
The **slope of the secant line** through and is the same as the average rate of change of over the interval , or the difference quotient:

$m_{sec} = \dfrac{f(x+h) - f(x)}{h}$



The **slope of the tangent line** through is the same as the instantaneous rate of change of at , or the **derivative**:

$m_{tan} = f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$



1. For a function $y = f(x)$, its **derivative** at *x* is the function f' defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists. If $f'(x)$ exists, then we say that *f* is **differentiable** at *x*. We sometimes call f' the **derived function**.

2. $f'(x)$ is the **slope of the tangent line** at the point .

3. For any function *f*, the **instantaneous rate of change** is equal to .

To Calculate the Derivative:

1) Write down the difference quotient, $\frac{f(x+h)-f(x)}{h}$
2) Simplify the difference quotient.
3) Evaluate the limit of the difference quotient as h approaches zero,

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}.$$

Example: Given $f(x)=3x^2-x$, find $f'(x)$.

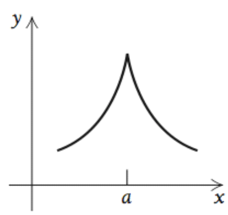
To Find the Equation of a Tangent Line at a Point $(a, f(a))$:

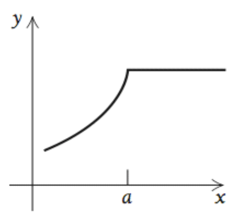
1) Find the derivative.
2) Find $f'(a)$. This represents the slope of the tangent line at $x = a$.
3) Substitute the slope, $f'(a)$, and the point $(a, f(a))$, into the point-slope or slope-intercept equations to find the equation of the

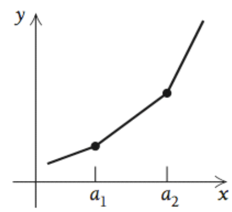
Given, $f(x) = 3x^2 - x$, find the equation of the tangent line at $x = -2$.

What does it look like where a function is ***not*** differentiable?

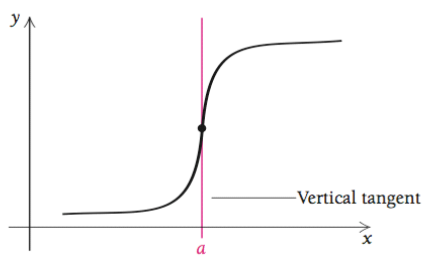
1) A function $f(x)$ is not differentiable at a point $x = a$, if there is a "corner" at a.







2) A function $f(x)$ is not differentiable at a point $x = a$, if there is a vertical tangent at a.



3) A function $f(x)$ is not differentiable at a point $x = a$, if it is not continuous at a.

Example: $g(x)$ is not continuous at –2, so $g(x)$ is not differentiable at $x = –2$.