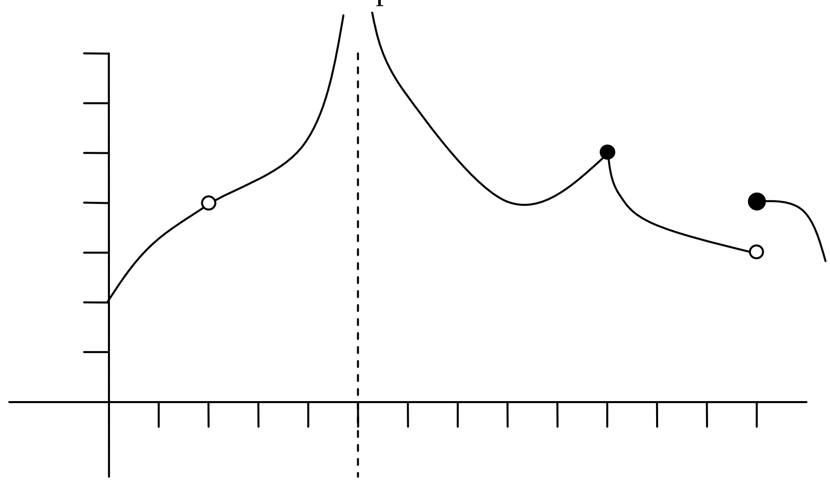
MA 160 FINAL EXAM REVIEW PROBLEMS

1. Use the information given about the function $f(x)$ in the sketch below to answer the questions. Assume that each tick mark represents one unit.



	Is $f(x)$ defined for this value of x? If so estimate $f(a)$.	Find $\lim_{x \to a} f(x)$ if it exists.	Is $f(x)$ continuous at this value of x?	Is $f(x)$ differentiable at this value of x?
$a=2$				
$a=5$				
$a=8$				
$a=10$				
$a=13$				

2. Use the definition of the derivative to find $f'(x)$ given that $f(x)=3x^2-x+5$.

3. Find the equation of the line tangent to the curve $y=\sqrt{x^2+16}$ when x = 3 on the curve.

4. Given $f(x)=\dfrac{x^2}{x-3}$

(a) Find $f'(x)$

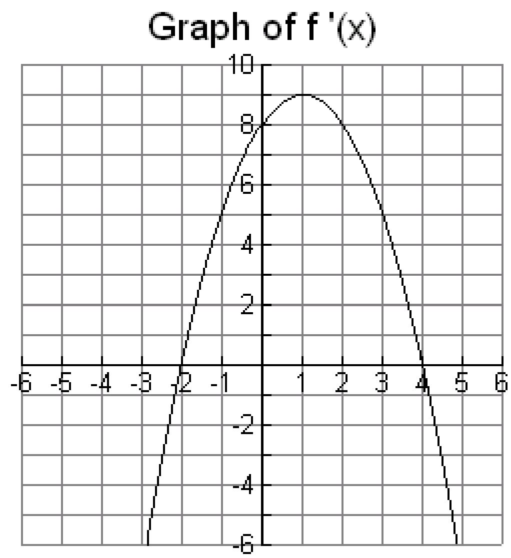
(b) Find all critical values for the function

(c) Determine whether the function has a relative maximum, a relative minimum, or neither at each critical value.

5. Differentiate the given functions:

(a) $f(x) = 9 - 4x^2 + 10x^{2/3}$	(f) $f(x) = \ln(x^2 - 2x + 5)^3$
(b) $f(x) = x^5(3x-2)^7$	(g) $f(x) = e^{x^2} - \sqrt{x}$
(c) $f(x) = \sqrt[3]{x^2 - 1}$	(h) $f(x) = \dfrac{e^x}{1+x^2}$
(d) $f(x) = \dfrac{1}{(x^2+x+1)^6}$	(i) $f(x) = 5x^2 e^{3x}$
(e) $f(x) = \left[\ln(x^2 - 2x + 5)\right]^3$	(j) $f(x) = \dfrac{x^2+3}{3x^2-5}$

6. The graph of $f'(x)$, the ***derivative*** of a function $f(x)$, is shown. Note that the graph of $f(x)$ is not given. Based on the graph of $f'(x)$, answer the following questions about $f(x)$.



(a) On what interval(s) is $f(x)$ increasing?
(b) On what interval(s) is $f(x)$ decreasing?
(c) At what value(s) of x does $f(x)$ have a relative maximum?
(d) At what value(s) of x does $f(x)$ have a relative minimum?
(e) On what interval(s) is $f(x)$ concave up?
(f) On what interval(s) is $f(x)$ concave down?
(g) At what value(s) of x does $f(x)$ have a point of inflection?

7. The temperature of a person during an illness is given by $F(t) = -0.1t^2 + 1.2t + 98.6$ where F is the temperature in degrees Fahrenheit at time t in days.
(a) Find the rate of change of the temperature with respect to time.
(b) Find $F(1.5)$ and write a sentence in everyday language explaining the meaning of your answer in the context of this situation.
(c) Find $F'(1.5)$ and write a sentence in everyday language explaining the meaning of your answer in the context of this situation.
(d) Why would the sign of $F'(t)$ be significant to a doctor?

8. The demand function for a certain product is $p(x) = 300\left(15 - \sqrt{x}\right)$.
(a) Find the revenue function.
(b) Find the marginal revenue when 64 items are sold.
(c) For what value of x is the revenue a maximum?

9. A manufacturer of cameras finds that the price at which it can sell x cameras per week is $p(x) = 500 - x$ dollars. The total cost of producing x cameras per week is $C(x) = 150 + 4x + x^2$ dollars.

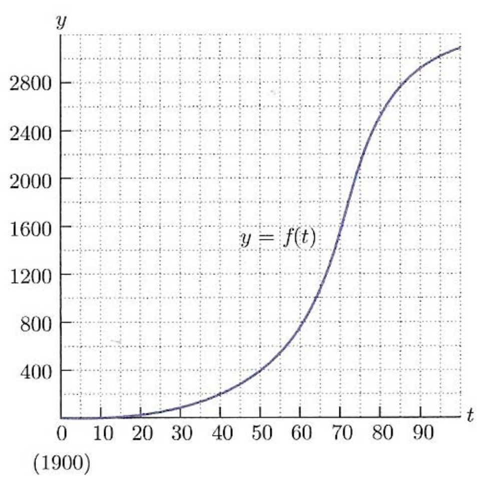
(a) Find the revenue function $R(x)$.

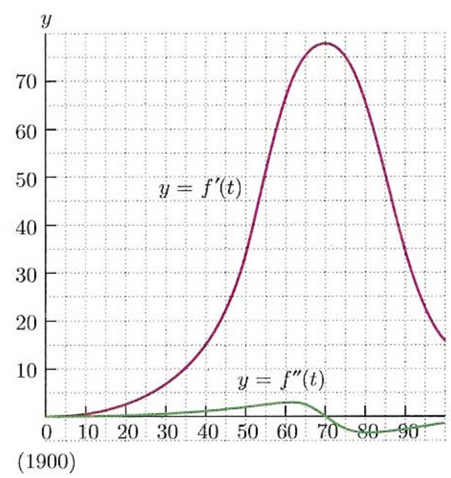
(b) Find the profit $P(x)$.

(c) Find the production level which maximizes the profit.

10. A tool rental company determines that it can achieve 500 daily rentals of jackhammers per year at a daily rental fee of $30. For each $1 increase in rental price, 10 fewer jackhammers will be rented. What rental price maximizes revenue?

11. U.S. electrical energy production (in trillions of kilowatt-hours) in year t (with 1900 corresponding to $t = 0$) is given by $f(t)$, where f and its derivatives are graphed below.





(a) How much electrical energy was produced in 1950?
(b) How fast was energy production rising in 1950?
(c) When did energy production reach 3000 trillion kilowatt-hours?
(d) When was the level of energy production rising at the rate of 10 trillion kilowatt-hours per year?
(e) When was energy production growing at the greatest rate? What was the level of production at that time?

12. Solve the given equations for x:

(a) $3^{x-1} = 9$ (b) $3e^{2x} = 21$ (c) $\ln(x-3) = 2$

13. Find the present value of $20,000 payable in five years if money is invested at 6% compounded continuously.

14. For what values of x is the function $y = e^{2x}\left(x^2 - 6\right)$ increasing? For what values of x is this function decreasing?

15. If the equation $y = 100e^{.03t}$ describes a certain population, where t is time in months, find
(a) The population when $t = 0$
(b) The time it takes the population to grow to twice its original number [this is the "doubling time"]
(c) The predicted population when $t = 70$ months and when $t = 120$ months.

16. The radioactive isotope iodine-131 has a half-life of 8 days.
(a) Find its decay constant.
(b) Find the amount remaining after 10 days if initially there is 5 mg.

17. Given $f(x) = 2x^3 - 3x^2 - 12x + 1$
(a) Find $f'(x)$ and $f''(x)$.
(b) Determine where $f(x)$ increases and where it decreases.
(c) Find any relative maximum or minimum point.
(d) Determine where $f(x)$ is concave up and concave down.
(e) Find any points of inflection.
(f) Sketch the graph of $f(x)$.

18. Suppose that $f(2) = 7,\ f'(2) = 0,\ and\ \ f''(2) = -4$. Describe the behavior of the function $f(x)$ at x = 2.

19. Find the antiderivative:
(a) $\int \left(x^3 - 6x^2 + 2x - 1\right)dx$ (b) $\int \frac{5}{x}dx$ (c) $\int (4 - 5e^{-5t} + \frac{e^{2t}}{3})\ dt$

20. Find the function that has derivative $f'(x) = 3x^2 + \frac{1}{x} - 4$ and whose graph contains the point (1,2).

21. Evaluate the given definite integrals:
(a) $\int_2^3 (6 + \frac{1}{x^2})dx$ (b) $\int_0^1 e^{4x}\ dx$ (c) $\int_1^4 \sqrt{x}\ dx$

22. Given the integral $\int_{-1}^{2} (x^2 + 4)\ dx$
(a) Calculate the area represented by the integral
(b) Sketch the area represented by the integral
(c) Find the average value of the function over the interval $-1 \le x \le 2$

23. Given the demand function $p = 1200 - 0.2x - 0.0001x^2$ find the consumer's surplus at x = 500.

24. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

25. Given $f(x) = x + \frac{4}{x}$

(a) Find $f'(x)$ and $f''(x)$.

(b) Use the results from part (a) to find the relative maximum and minimum points on the graph of $f(x)$.

(c) Sketch a graph of $f(x)$. Be sure to label the axes and indicate the scale used.

26. A rectangular garden of area 75 square feet is to be surrounded on three sides by a brick wall costing $10 per foot and on one side by a fence costing $5 per foot. Find the dimensions of the garden such that the cost of the materials is minimized.

27. Find the area bounded by the curves $y = x + 1$ and $y = x^2 - 3x - 4$.

28. Given $f(x, y) = 4xy^2 - 3x^3 y + y^5$, find:

(a) $\frac{\partial f}{\partial x}(x, y)$ (b) $\frac{\partial f}{\partial y}(x, y)$

29. Use the appropriate features of your graphing calculator to find the following. Round your answers to the nearest thousandth [three decimal places].

(a) Find the zero(s) of the function $f(x) = \sqrt{x+2} - x + 2$.

(b) Find the zero(s) of the function $f(x) = \frac{x}{x+2} - x^2 + 1$.

(c) Find the point(s) of intersection of the graphs of the functions $f(x) = e^{2x}$ and $g(x) = 4 - x^2$.

(d) Find the point(s) of intersection of the graphs of the functions $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x^2 - 1}$.

(e) Given $f(x) = 8^{2x+3}$, find $f'(0)$.

(f) Given $f(x) = \frac{\sqrt{x^2+1}}{x-8}$, find $f'(2)$.

(g) Find the relative extremes and inflection points on the graph of $f(x) = 3x^5 - 20x^3 - 120x$

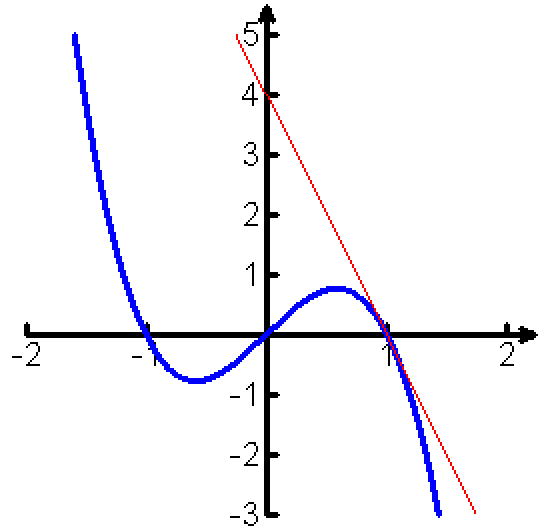
Hint: window setting [-1, 4] by [-325, 325]

(h) Calculate $\int_1^4 \ln x dx$.

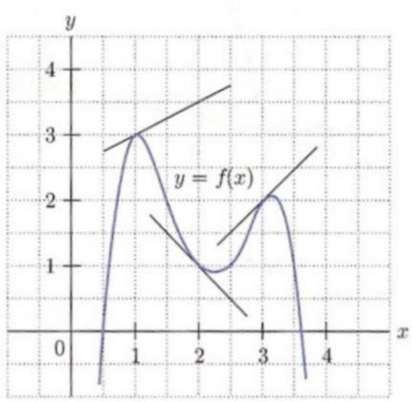
(i) Calculate $\int_0^{0.5} \frac{1}{1+x^2} dx$

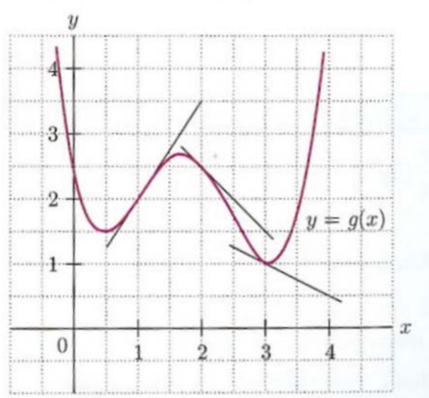
30. The population of a country was 4.5 million in 1987 ($t = 0$) and 6.4 million in 1994. Assume that the population is growing at a rate proportional to its size. What was the average population between 1987 and 2007?

31. Use the graph of f [shown in blue] to find the equation of the tangent line at $x = 1$. Write in slope-intercept form.



32. Use the graphs of $f(x)$ and $g(x)$ to find $h'(1)$ for the following.





(a) $h(x) = f(x)g(x)$

(b) $h(x) = \dfrac{f(x)}{g(x)}$

(c) $h(x) = [f(x)]^2$

(d) $h(x) = f(g(x))$

(e) $h(x) = g(f(x))$

(f) $h(x) = f(x^2) + 3x$

ANSWERS

1. At $a=2$: $f(2)$ undefined, limit = 4, discontinuous, not differentiable
   At $a=5$: $f(5)$ undefined, no limit, discontinuous, not differentiable
   At $a=8$: $f(8)=4$, limit = 4, continuous, differentiable
   At $a=10$: $f(10)=5$, limit = 5, continuous, not differentiable
   At $a=13$: $f(13)=4$, no limit, discontinuous, not differentiable
2. $f'(x)=6x-1$
3. $y=\frac{3}{5}x+\frac{16}{5}$
4. (a) $f'(x)=\frac{x(x-6)}{(x-3)^2}$
   (b) critical values are $x=0$ and $x=6$
   (c) rel. max. at $(0,0)$; rel. min. at $(6,12)$

5.

(a) $f'(x)=-8x+\frac{20}{3}x^{-\frac{1}{3}}$	(f) $f'(x)=\frac{6(x-1)}{x^2-2x+5}$
(b) $5x^4(3x-2)^7+21x^5(3x-2)^6$ $=2x^4(3x-2)^6(18x-5)$	(g) $f'(x)=2xe^{x^2}-\frac{1}{2}x^{-1/2}$
(c) $f'(x)=\frac{2x}{3(x^2-1)^{2/3}}$	(h) $f'(x)=\frac{e^x(x-1)^2}{(1+x^2)^2}$
(d) $f'(x)=\frac{-6(2x+1)}{(x^2+x+1)^7}$	(i) $f'(x)=5xe^{3x}(3x+2)=15x^2e^{3x}+10xe^{3x}$
(e) $f'(x)=\frac{6(x-1)}{x^2-2x+5}\cdot\left[\ln\left(x^2-2x+5\right)\right]^2$	(j) $f'(x)=-\frac{28x}{(3x^2-5)^2}$

6. (a) (-2, 4) (b) $(-\infty,-2)$ *and* $(4,\infty)$ (c) x = 4 (d) x = -2
   (e) $(-\infty,1)$ (f) $(1,\infty)$ (g) $x=1$

7. (a) $F'(t)=-0.2t+1.2$
(b) $\approx 100.2^\circ$ After one and a half days, the person's temperature is about 100.2° F.
(c) 0.9° *per day*
After one and a half days, the person's temperature is rising at a rate of about 0.9° *per day*.

8. (a) $R(x)=300x\left(15-\sqrt{x}\right)$ (b) $R'(64)=\$900$ per item
   (c) $x=100$ items

9. (a) $R(x) = 500x - x^2$ (b) $P(x) = 496x - 2x^2 - 150$ (c) $x = 124$

10. \$40

11.
a. Since $f(50) = 400$, the amount of energy produced was 400 trillion kilowatt-hours.

b. Since $f'(50) = 35$, the rate of change was 35 trillion kilowatt-hours per year.

c. Since $f(t) = 3000$ at $t = 95$, the production level reached 300 trillion kilowatt-hours in 1995.

d. Since $f'(t) = 10$ at $t = 35$, the production level was rising at the rate of 10 trillion kilowatt-hours per year in 1935.

e. Looking at the graph of $y = f'(t)$, the value of $f'(t)$ appears to be greatest at $t = 70$. To confirm, observe that the graph of $y = f''(t)$ crosses the t-axis at $t = 70$. Energy production was growing at the greatest rate in 1970. Since $f(70) = 1600$, the production level at that time was 1600 trillion kilowatt-hours.

12. (a) $x = 3$
(b) $x = \dfrac{\ln 7}{2}$
(c) $x = 3 + e^2$

13. $\approx \$14{,}815$

14. y is increasing for $x < -3$ and for $x > 2$;
y is decreasing for $-3 < x < 2$

15. (a) 100 (b) 23 months
(c) 817 and 3660

16. (a) .0866 (b) = 2.1 mg

17. (a) $f'(x) = 6(x-2)(x+1)$, $f''(x) = 6(2x-1)$

(b) $x < -1$ inc. (d) $x < 1/2$ down
$-1 < x < 2$ dec. $x > 1/2$ up
$x > 2$ inc.

(c) $(-1, 8)$ relative maximum, $(2, -19)$ relative minimum

(e) $\left(1/2, -5\,1/2\right)$ point of inflection

18. The function $f(x)$ has a relative maximum at the point (2, 7).

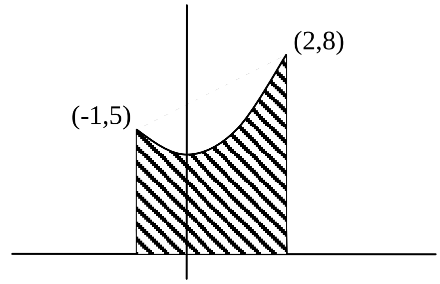
19. (a) $\dfrac{1}{4}x^4 - 2x^3 + x^2 - x + C$ (b) $5\ln|x| + C$

(c) $4t + e^{-5t} + \dfrac{e^{2t}}{6} + C$

20. $f(x) = x^3 + \ln x - 4x + 5$

21. (a) $6\dfrac{1}{6} = \dfrac{37}{6}$ (b) $\dfrac{1}{4}e^4 - \dfrac{1}{4}$ (c) $\dfrac{14}{3}$

22. (a) 15 (b)
(c) 5



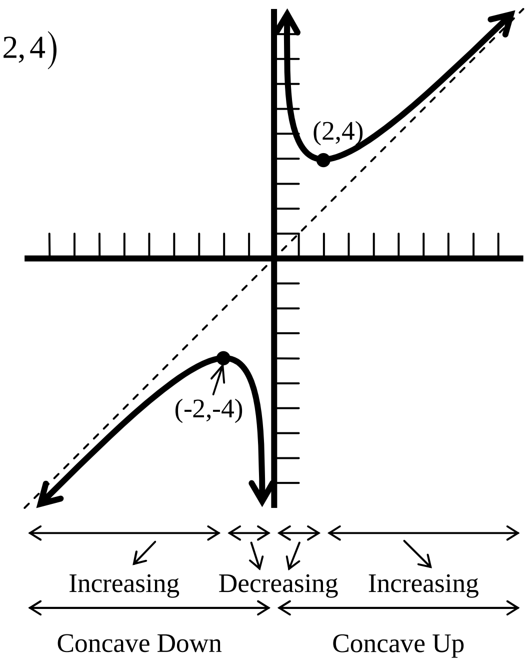
23. \$33,333.33

24. 1800 liters

25. (a) $f'(x) = 1 - \frac{4}{x^2}$
$f''(x) = \frac{8}{x^3}$

(b) rel max $(-2,-4)$
rel min at $(2,4)$

(c)



26. 7.5 ft. by 10 ft.

27. 36 sq. units

28. (a) $4y^2 - 9x^2 y$ (b) $8xy - 3x^3 + 5y^4$

29. (a) 4.562 (b) $\{-2.481, -0.689, 1.170\}$ (c) $\{-1.995, 0.639\}$ (d) 1.272
(e) 2129.354 (f) -0.211 (g) rel min: $f(2.338) \approx -326.585$
rel. max: $f(-2.338) \approx 326.585$, inflection points: $(0,0)$, $(1.414, -209.304)$, $(-1.414, 209.304)$ (h) 2.545 (i) 0.464

30. $y = -4x + 4$

31. $\approx$ 7.76 million

32. (a) $\frac{11}{2}$ (b) $-\frac{7}{8}$ (c) 3 (d) $-\frac{3}{2}$ (e) $-\frac{1}{4}$ (f) 4