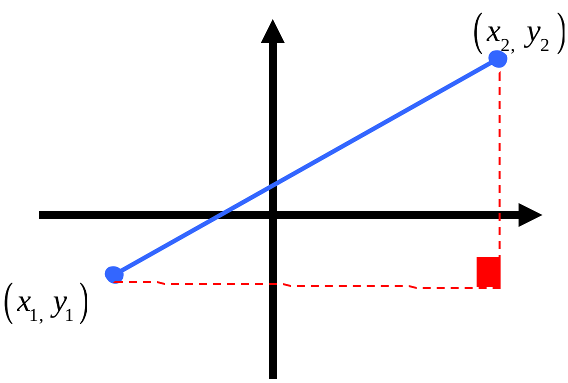
Distance and Midpoint Formulas (1.1)

Objectives:

1. Use the Distance Formula (p. 4)
2. Use the Midpoint Formula (p. 7)
3. Graph Equations by Hand by Plotting Points (p. 7)
4. Graph Equations Using a Graphing Utility (p. 10)
5. Use a Graphing Utility to Create Tables (p. 12)
6. Find Intercepts from a Graph (p. 12)
7. Use a Graphing Utility to Approximate Intercepts (p. 13)

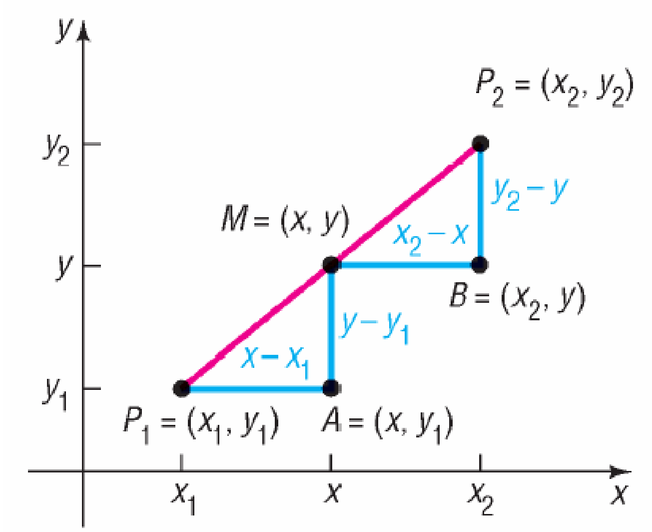
Find the distance between the points, $(x_{1,}\ y_1)$ and $(x_{2,}\ y_2)$



Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

1. Determine the distance $d(P_1, P_2)$ between the points $P_1(1.4, 3.2)$ and $P_2(-.4, 1.7)$. Round to the nearest hundredth.



Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

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2. Find the midpoint of the line segment joining the points $P_1(1.4, 3.2)$ and $P_2(-.4, 1.7)$.

Intercepts, Symmetry, Graphing, & Key Equations (1.2)

<u>Procedure for Finding Intercepts</u>

- To find the ***x*-intercept(s),** if any, of the graph of an equation, let $y = 0$ in the equation and solve for x. (x-intercept(s) are also called **zeroes** or **roots** of the equation.) [**ZERO/ROOT** feature on the graphing calculator]
- To find the ***y*-intercept(s)**, if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

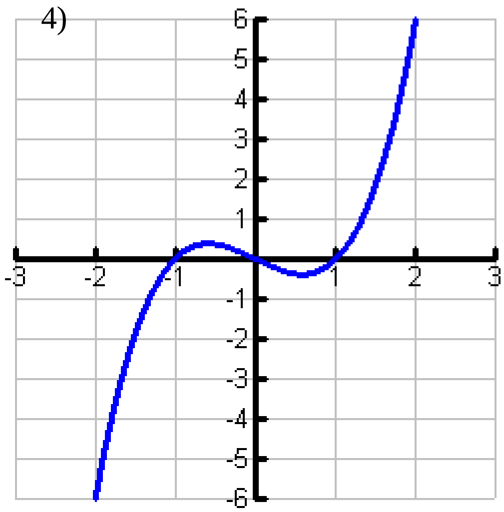
Part 1: Use a graphing utility to graph the equation and identify any x- or y-intercepts. Round your answer to the nearest hundredth where appropriate..

1) $y = 3 - \frac{3}{2}|x|$

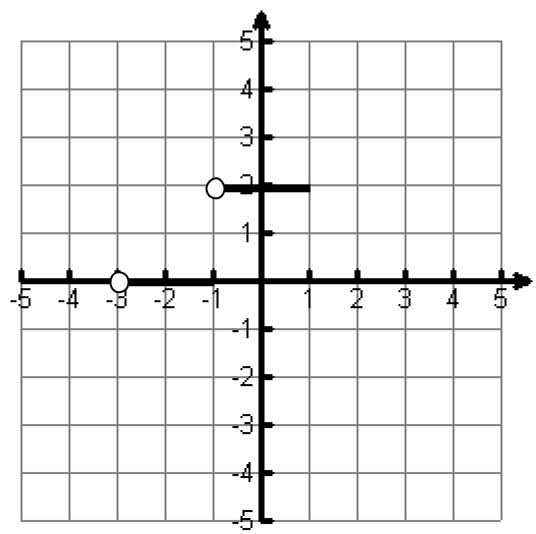
2) $y = x^4 + 3x^2 - x - 5$

Part 2: Find the intercepts of the graphs.

3)



4)

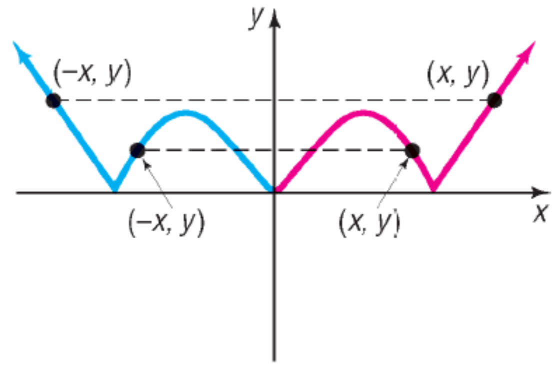


<u>Steps for Graphing an Equation Using a Graphing Utility</u>

- **Step 1**: Solve the equation for y in terms of x.
- **Step 2**: Get into the graphing mode of your graphing utility and enter the result found in Step1.
- **Step 3**: Select the viewing window. (It is common to start with the standard viewing window initially.)
- **Step 4:** Graph.
- **Step 5:** Adjust the viewing window.

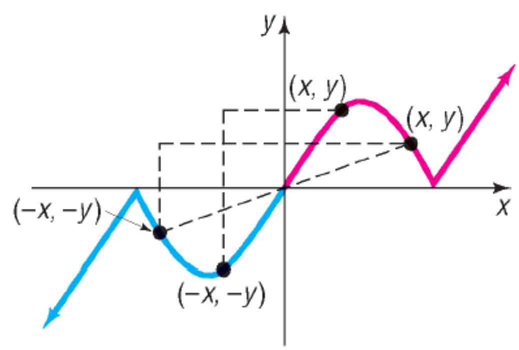
Symmetry

- A graph is said to be **symmetric with respect to the *y*-axis** if for every point (x,y) on the graph, the point $(-x,y)$ is on the graph.



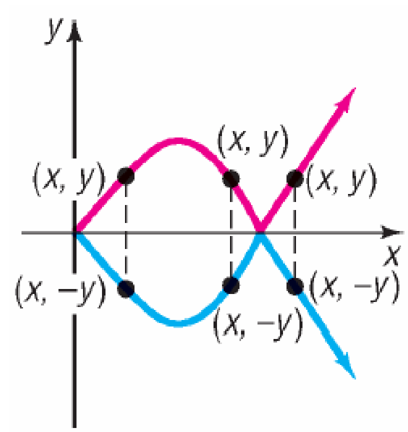
Symmetry with respect to the *y*-axis

- A graph is said to be **symmetric with respect to the origin** if for every point (x,y) on the graph, the point $(-x,-y)$ is on the graph.



Symmetry with respect to the origin

- A graph is said to be **symmetric with respect to the *x*-axis** if for every point (x,y) on the graph, the point $(x,-y)$ is on the graph.



Symmetry with respect to the *x*-axis

EXAMPLE Symmetric Points

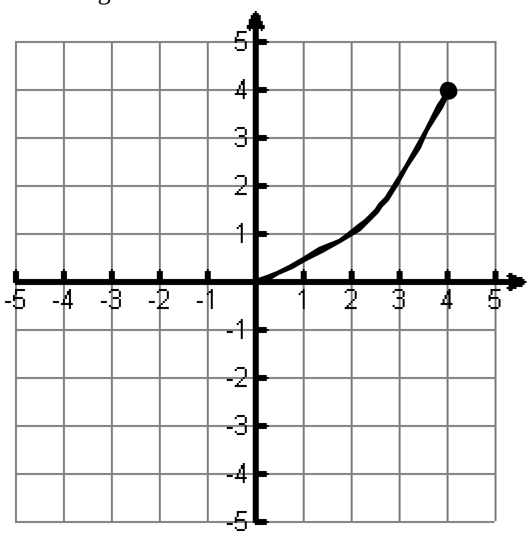
(a) If a graph is symmetric with respect to the *x*-axis and the point (-2, 3) is on the graph, then what point is also on the graph?

(b) If a graph is symmetric with respect to the *y*-axis and the point (-1, 3) is on the graph, then what point is also on the graph?

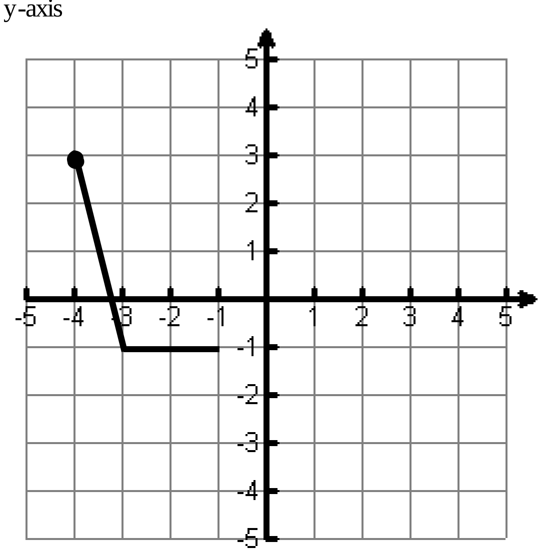
(c) If a graph is symmetric with respect to the origin and the point (-1, 3) is on the graph, then what point is also on the graph?

Draw a complete graph so that it has the symmetry indicated.

a. Origin



b. y-axis



Tests for Symmetry

To test the graph of an equation for symmetry with respect to the

x-Axis Replace y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.

y-Axis Replace x by $-x$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the y-axis.

Origin Replace x by $-x$ and y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.

Solving Equations (1.3 and A.5)

Steps for Approximating Solutions of Equations Using Zero (or Root)

- Write the equation in the form {expression in x } = 0
- Graph Y_1= {expression in x }.
- Use ZERO (or ROOT) to determine each x-intercept of the graph.

Steps for Approximating Solutions of Equations Using Intersect

- Graph Y_1={expression in x on the left hand side of equation}.
- Graph Y_2={expression in x on the right hand side of equation}.
- Use INTERSECT to determine each x-coordinate of the points of intersection.

Methods for Solving Quadratic Equations

- Factoring
- Graphing
- Square Root Method
- Completing the Square Method
- Quadratic Formula

Method for Solving Absolute Value Equations

Rewrite the equation without the absolute value symbol using the definition of absolute value.

$$|x| = \begin{cases} \\ \\ \end{cases}$$

Solve algebraically. Verify with your graphing utility.

a. $|2x-1|=13$

b. $|x^2 - 2x| = 3x - 6$

Methods for Solving Equations Containing Radicals

1) Isolate the most complicated radical on one side of the equation.
2) Eliminate radical by raising each side to a power equal to the index of the radical. (Note: Steps 1 and 2 may need to be repeated if the equation contains more than one radical.)
3) Solve resulting equation.
4) Check for extraneous solutions.

a. $\sqrt[4]{2x+21}-3=0$

b. $\sqrt{2x-1}+\sqrt{x-4}=4$

Methods for Solving Rational Equations

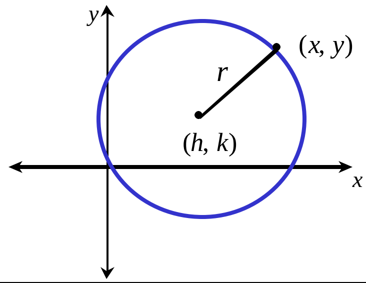
1. Factor, if possible, the denominators.
2. Find the LCD of the rational expressions.
3. Multiply both sides of the equation by the LCD.
4. Solve the resulting simpler equation.
5. Check that each result satisfies the original equation. Results that do not satisfy the original equation are called ***extraneous solutions***.

a. $\frac{3}{2}-\frac{1}{x-4}=\frac{-2}{2x-8}$

b. $\frac{x+1}{2x+6}=\frac{x}{x-3}-\frac{x-1}{x^2-9}$

Circles (1.5)

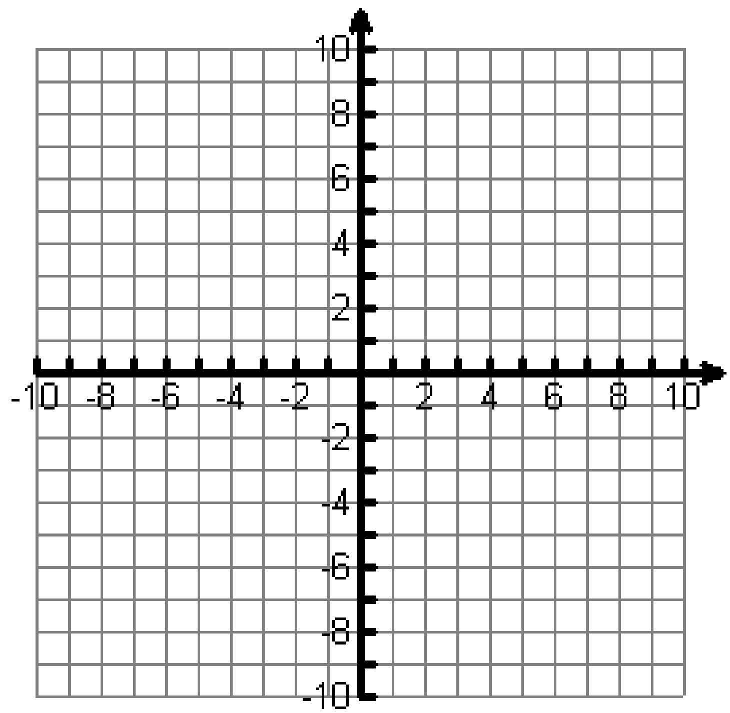
A **circle** is a set of points in the *xy*-plane that are a fixed distance r from a fixed point (h, k). The fixed distance r is called the **radius**, and the fixed point (h, k) is called the **center** of the circle.



The **standard form of an equation of a circle** with radius r and center (h, k) is

$$(x-h)^2+(y-k)^2=r^2$$

Graph $(x+1)^2+(y-3)^2=16$



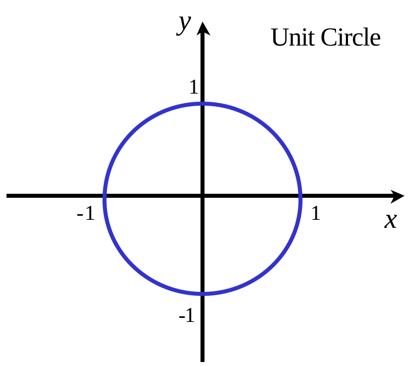
Graph $(x+1)^2+(y-3)^2=16$ using a graphing utility.

The standard form of an equation of a circle of radius *r* with center at the origin (0, 0) is

$$x^2 + y^2 = r^2$$

If the radius of a circle whose center is at the origin is $r = 1$, then we have a **unit circle** whose equation is of the form

$$x^2 + y^2 = 1$$



The **general form of the equation of a circle** is

$$x^2 + y^2 + ax + by + c = 0$$

Find the center and radius of $x^2 + y^2 - 4x + 8y - 5 = 0$

Completing the Square

I. What number must be added to each expression below to complete the square?

1. $x^2 + 2x$ 2. $x^2 - 8x$ 3. $x^2 - 5x$

4. $x^2 + x$ 5. $x^2 + \frac{2}{3}x$ 6. $x^2 - \frac{5}{7}x$

II. Write each of the above expressions in the form of $(x-h)^2 + k$

III. Write each of the following in the form $a\left(x^2 + rx\right) + c$

1. $5x^2 - 10x + 3$ 2. $2x^2 - 6x + 4$

3. $5x^2 + 15x + 7$ 4. $5x^2 - 7x + 2$

IV. Write the expressions in part III in the form of $a(x-h)^2 + c$

Answers:

I. 1. 1 2. 16 3. $\frac{25}{4}$ 4. $\frac{1}{4}$ 5. $\frac{1}{9}$ 6. $\frac{25}{196}$

II. 1. $(x+1)^2 - 1$ 2. $(x-4)^2 - 16$ 3. $\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}$ 4. $\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$ 5. $\left(x - \frac{1}{3}\right)^2 - \frac{1}{9}$ 6. $\left(x - \frac{5}{14}\right)^2 - \frac{25}{196}$

III. 1. $5\left(x^2 - 2x\right) + 3$ 2. $2\left(x^2 - 3x\right) + 4$ 3. $5\left(x^2 + 3x\right) + 7$ 4. $5\left(x^2 - \frac{7}{5}x\right) + 2$

Almost answers: IV. 1. $5(x-1)^2 + 3 - 5$ 2. $2\left(x - \frac{3}{2}\right)^2 + 4 - (2)\left(\frac{3}{2}\right)^2$

3. $5\left(x + \frac{3}{2}\right)^2 + 7 - (5)\left(\frac{3}{2}\right)^2$ 4. $5\left(x + \frac{7}{10}\right)^2 + 2 - (5)\left(\frac{7}{10}\right)^2$