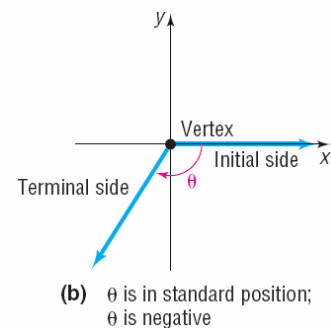
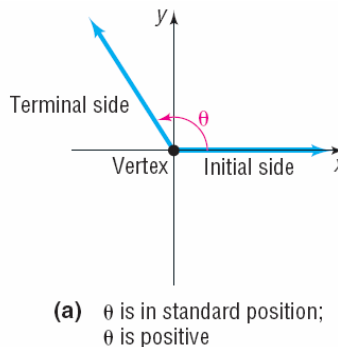
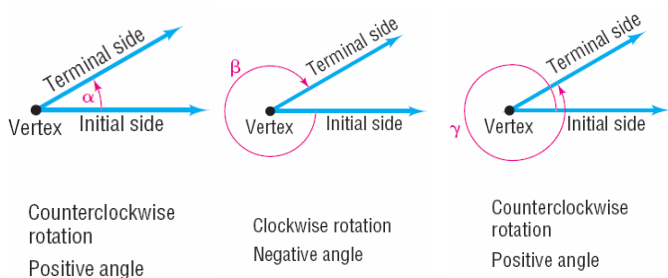


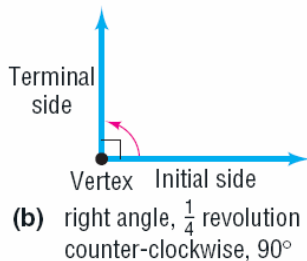
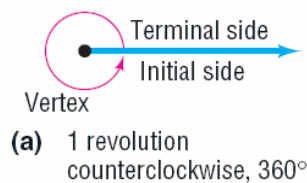
Section 6.1: Angles and Their Measure

**Learning Objectives:**

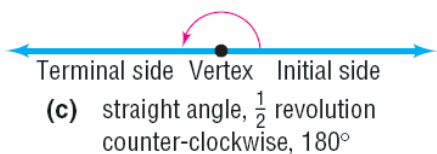
1. Convert between Decimals and Degrees, Minutes, Seconds Forms of Angles (p. 354)
2. Find the Arc Length of a Circle (p. 356)
3. Convert from Degrees to Radians and from Radian to Degrees (p. 356)
4. Find the Area of a Sector of a Circle (p. 359)
5. Find the Linear Speed of an Object Traveling in Circular Motion (p. 360)



**Degrees**

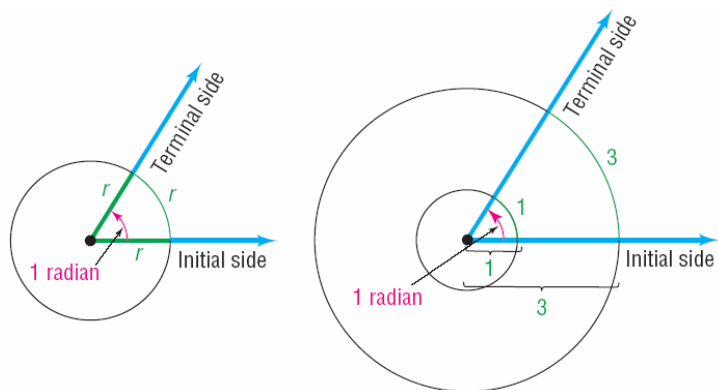


1 counterclockwise revolution =  $360^\circ$   
 $1^\circ = 60'$      $1' = 60''$

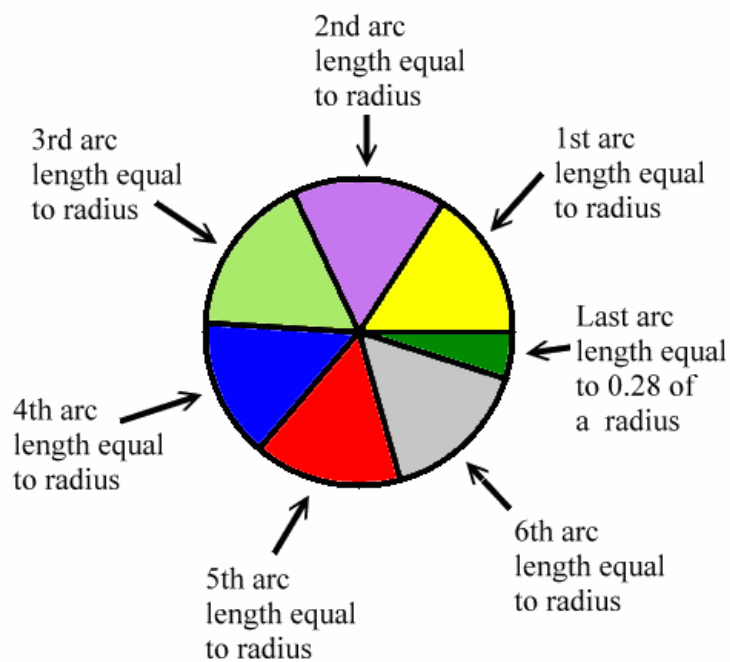


1. Convert  $8732'15''$  to a decimal in degrees.

## Radians



<http://id.mind.net/~zona/mmts/trigonometryRealms/radianDemo1/RadianDemo1.html>



2. Convert  $-\frac{11\pi}{3}$  to degree measure.

Section 6.2: Trigonometric Functions: Unit Circle Approach

**Learning Objectives:**

1. Find the Exact Values of the Trigonometric Functions Using a Point on the Unit Circle (p. 367)
2. Find the Exact Values of the Trigonometric Functions of Quadrantal Angles (p. 368)
3. Find the Exact Values of the Trigonometric Functions of  $\frac{p}{4} = 45^\circ$  (p. 370)
4. Find the Exact Values of the Trigonometric Functions of  $\frac{p}{6} = 30^\circ$  and  $\frac{p}{3} = 60^\circ$  (p. 371)
5. Find the Exact values of the Trigonometric Functions for Integer Multiples of  $\frac{p}{6} = 30^\circ$ ,  $\frac{p}{4} = 45^\circ$ , and  $\frac{p}{3} = 60^\circ$  (p. 374)
6. Use a Calculator to Approximate the Values of the Trigonometric Functions of Acute Angles (p. 375)
7. Use a Circle of Radius  $r$  to Evaluate the Trigonometric Functions (p. 376)

Let  $t$  be a real number and let  $P = (x, y)$  be the point on the unit circle that corresponds to  $t$ .

The **sine function** associates with  $t$  the  $y$ -coordinate of  $P$  and is denoted by

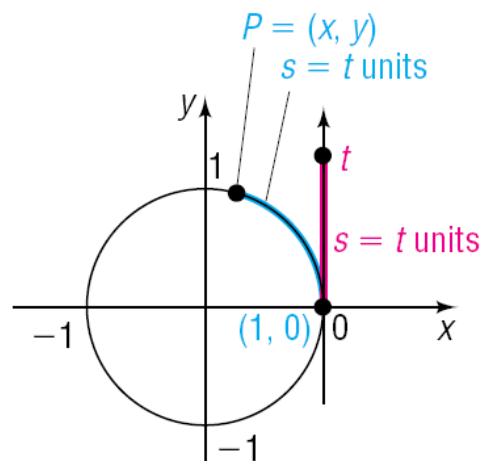
$$\sin t = y$$

The **cosine function** associates with  $t$  the  $x$ -coordinate of  $P$  and is denoted by

$$\cos t = x$$

If  $x \neq 0$ , the **tangent function** associates with  $t$  the ratio of the  $y$ -coordinate to the  $x$ -coordinate of  $P$  and is denoted by

$$\tan t = \frac{y}{x}$$



Let  $t$  be a real number and let  $P = (x, y)$  be the point on the unit circle that corresponds to  $t$ .

If  $y \neq 0$ , the **cosecant function** is defined as

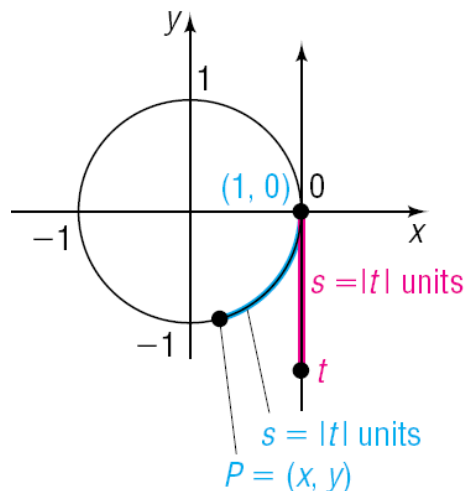
$$\csc t = \frac{1}{y}$$

If  $x \neq 0$ , the **secant function** is defined as

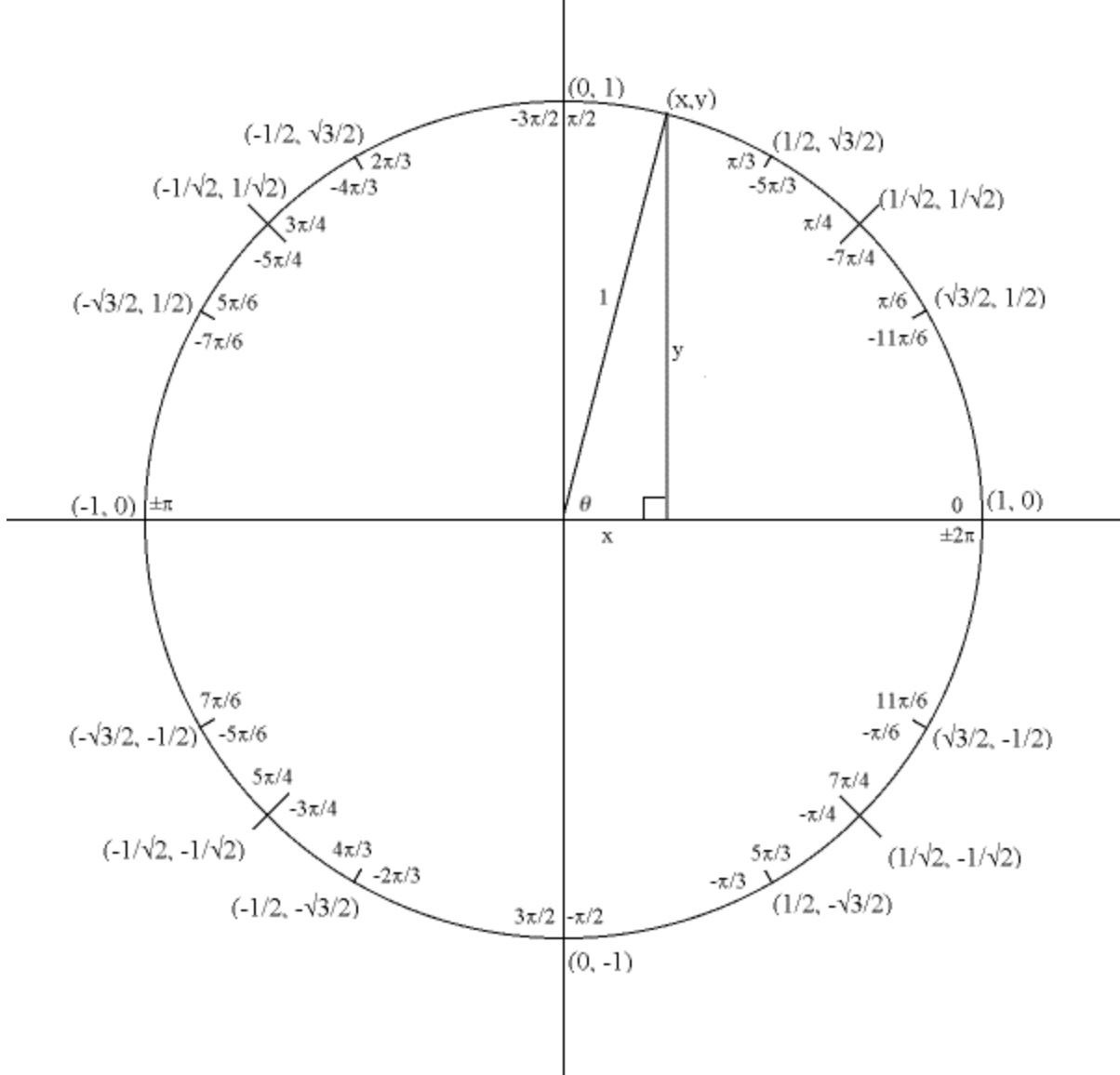
$$\sec t = \frac{1}{x}$$

If  $y \neq 0$ , the **cotangent function** is defined as

$$\cot t = \frac{x}{y}$$

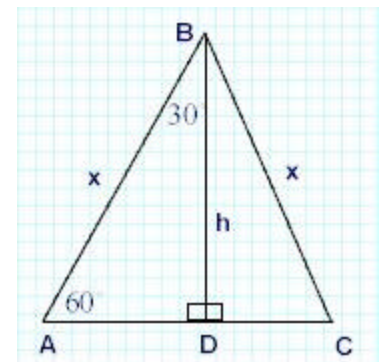


# The Unit Circle: $x^2 + y^2 = 1$



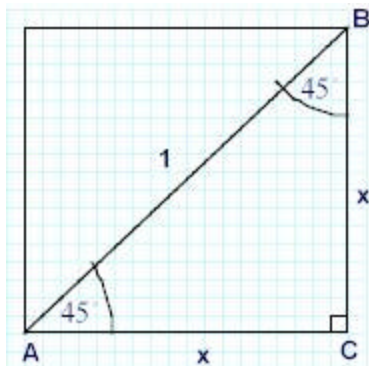
## 30°-60°-90°

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is  $\sqrt{3}$  times as long as the shorter leg.



In a 45°-45°-90° triangle, the hypotenuse is  $\sqrt{2}$  times as long as each leg.

## 45°-45°-90°



**Sine Function:**  $f(x) = \sin x$

$x$	$\sin x$
$0$	
$\frac{p}{2}$	
$p$	
$\frac{3p}{2}$	
$2p$	

Domain:

Range:

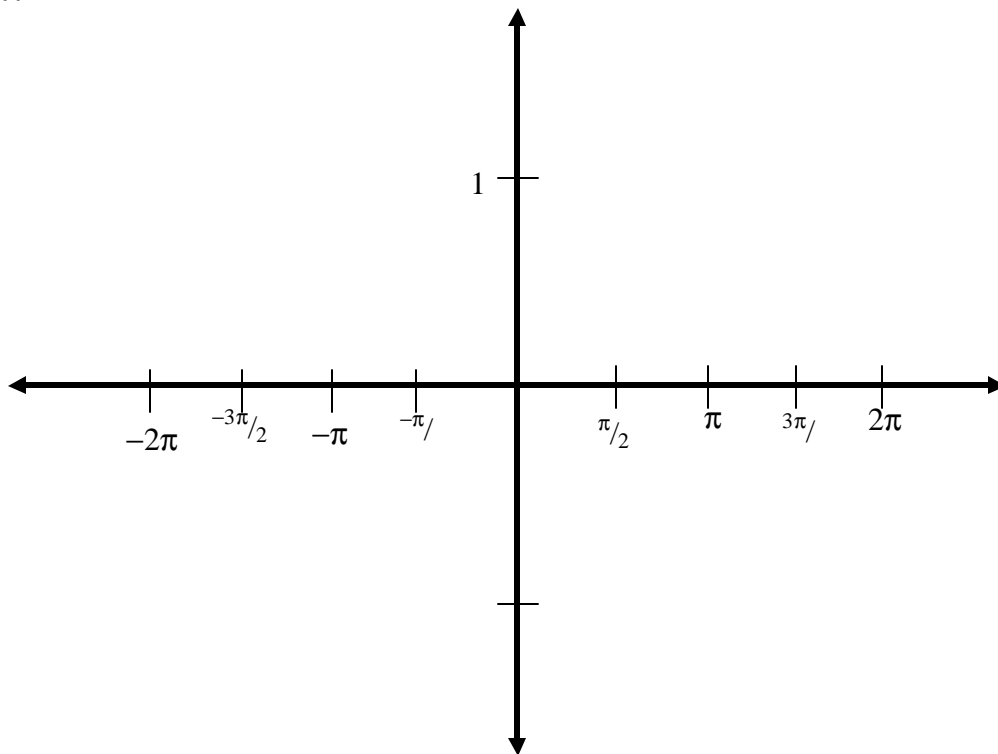
Symmetry:

Period:

x-intercepts:

y-intercept:

relative extrema



**Cosine Function:**  $f(x) = \cos x$

$x$	$\cos x$
$0$	
$\frac{p}{2}$	
$p$	
$\frac{3p}{2}$	
$2p$	

Domain:

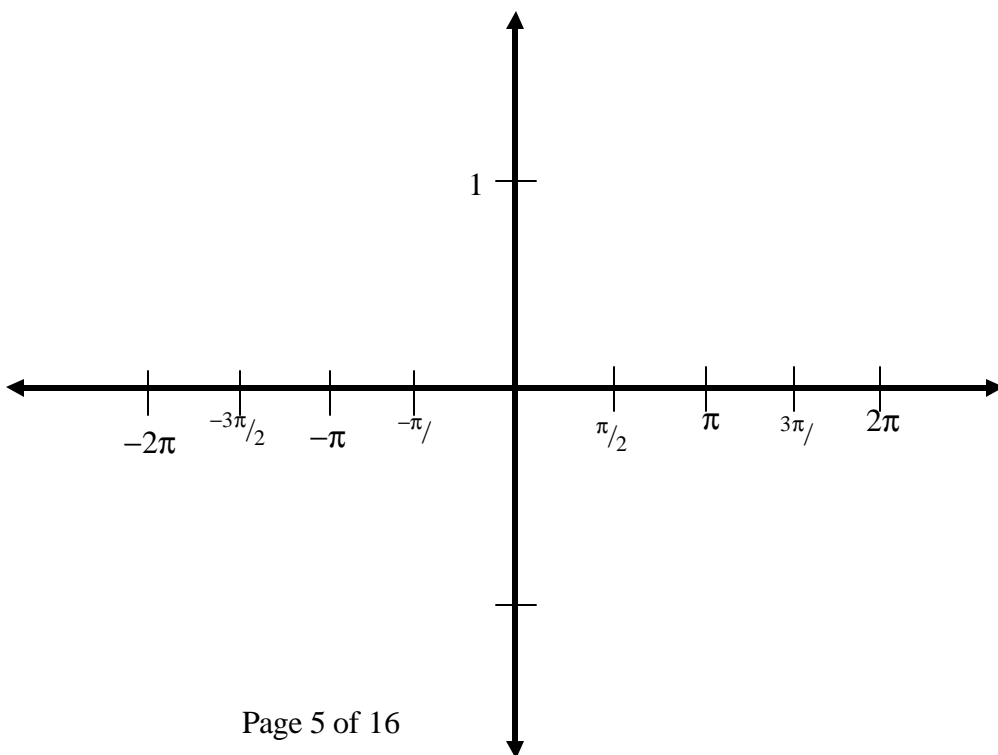
Range:

Symmetry:

Period:

x-intercepts:

y-intercept:

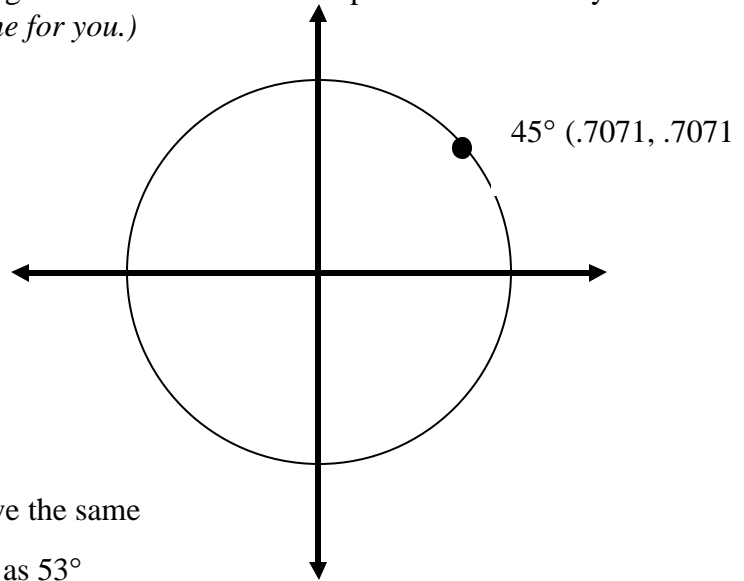


For an angle  $\theta$  in standard position, let  $P = (x, y)$  be the point on the terminal side of  $\theta$  that is also on the circle  $x^2 + y^2 = r^2$ . Then

$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}, x \neq 0$
$\csc \theta = \frac{r}{y}, y \neq 0$	$\sec \theta = \frac{r}{x}, x \neq 0$	$\cot \theta = \frac{x}{y}, y \neq 0$

1. Mark the following angles on the unit circle below **and** give the coordinates of the point determined by each angle to four decimal places. (An example has been done for you.)

- a.  $100^\circ$
- b.  $-7$  radians
- c.  $-200^\circ$
- d.  $3$  radians
- e.  $-720^\circ$



2. Find **two** angles, one positive and one negative, that have the same

- a. sine as  $-\frac{5p}{3}$
- b. cosine as  $53^\circ$

3. Give the exact value of the trigonometric functions if: (*Hint: Use the fact that the trigonometric functions are periodic. Do not use a calculator.*)

- a.  $t = \frac{13p}{6}$
- b.  $t = -9p$

$$\sin(\theta + 2\pi k) = \sin \theta \quad \cos(\theta + 2\pi k) = \cos \theta$$

where  $k$  is any integer

A function  $f$  is called **periodic** if there is a positive number  $p$  such that, whenever  $\theta$  is in the domain of  $f$ , so is  $\theta + p$ , and

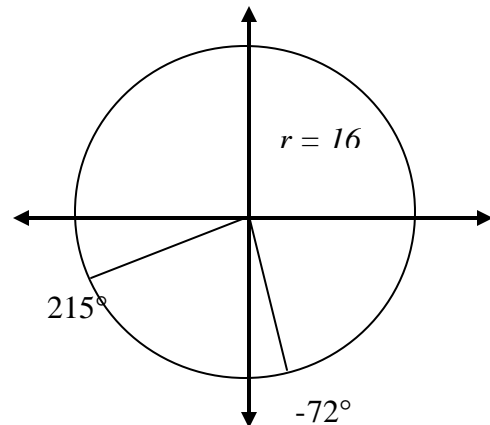
$$f(\theta + p) = f(\theta)$$

If there is a smallest such number  $p$ , this smallest value is called the **(fundamental) period** of  $f$ .

4. Evaluate. Round your answer to four decimal places. (Check your calculator mode!)

- a.  $\sec(-5.42)$
- b.  $\csc\left(\frac{p}{3}\right)$
- c.  $\sin(76.4^\circ)$
- d.  $\cos(35^\circ 18')$

5. Find approximations to two decimal places for the coordinates of points A and B.





## Fundamental Trigonometric Identities

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \\ \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \sin^2 \theta + \cos^2 \theta &= 1 & \tan^2 \theta + 1 &= \sec^2 \theta & \cot^2 \theta + 1 &= \csc^2 \theta\end{aligned}$$

### Finding the Values of the Trigonometric Functions When One Is Known

Given the value of one trigonometric function and the quadrant in which  $\theta$  lies, the exact value of each of the remaining five trigonometric functions can be found in either of two ways.

#### Method 1 Using a Circle of Radius $r$

**STEP 1:** Draw a circle showing the location of the angle  $\theta$  and the point  $P = (x, y)$  that corresponds to  $\theta$ . The radius of the circle is  $r = \sqrt{x^2 + y^2}$ .

**STEP 2:** Assign a value to two of the three variables  $x, y, r$  based on the value of the given trigonometric function and the location of  $P$ .

**STEP 3:** Use the fact that  $P$  lies on the circle  $x^2 + y^2 = r^2$  to find the value of the missing variable.

**STEP 4:** Apply the theorem on page 382 to find the values of the remaining trigonometric functions.

#### Method 2 Using Identities

Use appropriately selected identities to find the value of each of the remaining trigonometric functions.

8. Find the values of the remaining trigonometric functions given: (*Use your Fundamental Trig Identities.*)

a.  $\tan \mathbf{b} = 5$       and  $\cos \mathbf{b} < 0$                       b.  $\cos \mathbf{a} = \frac{1}{4}$       and  $\sin \mathbf{a} < 0$

**Tangent Function:**  $f(x) = \tan x$

$x$	$\tan x$
$0$	
$\frac{p}{2}$	
$p$	
$\frac{3p}{2}$	
$2p$	

Domain:

Range:

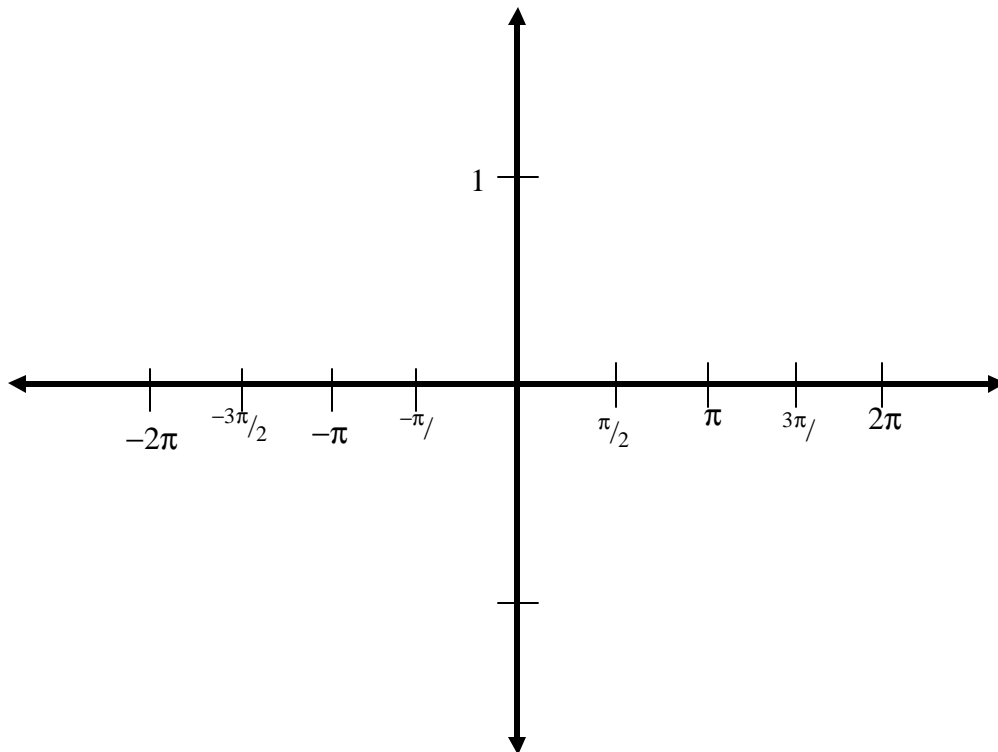
Vertical Asymptote(s):

Symmetry:

Period:

x-intercepts:

y-intercept:



**Cotangent Function:**  $f(x) = \cot x$

$x$	$\cot x$
$0$	
$\frac{p}{2}$	
$p$	
$\frac{3p}{2}$	
$2p$	

Domain:

Range:

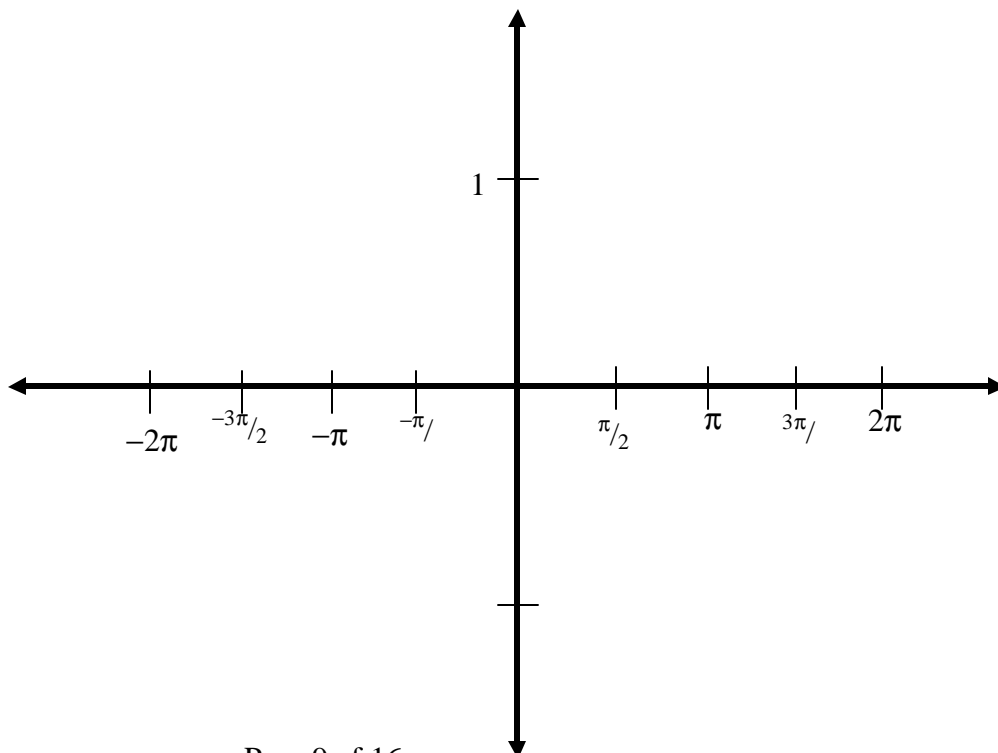
Vertical Asymptote(s):

Symmetry:

Period:

x-intercepts:

y-intercept:



**Secant Function:**  $f(x) = \sec x$

$x$	$\cos x$	$\sec x$
$0$		
$\frac{p}{2}$		
$p$		
$\frac{3p}{2}$		
$2p$		

Domain:

Range:

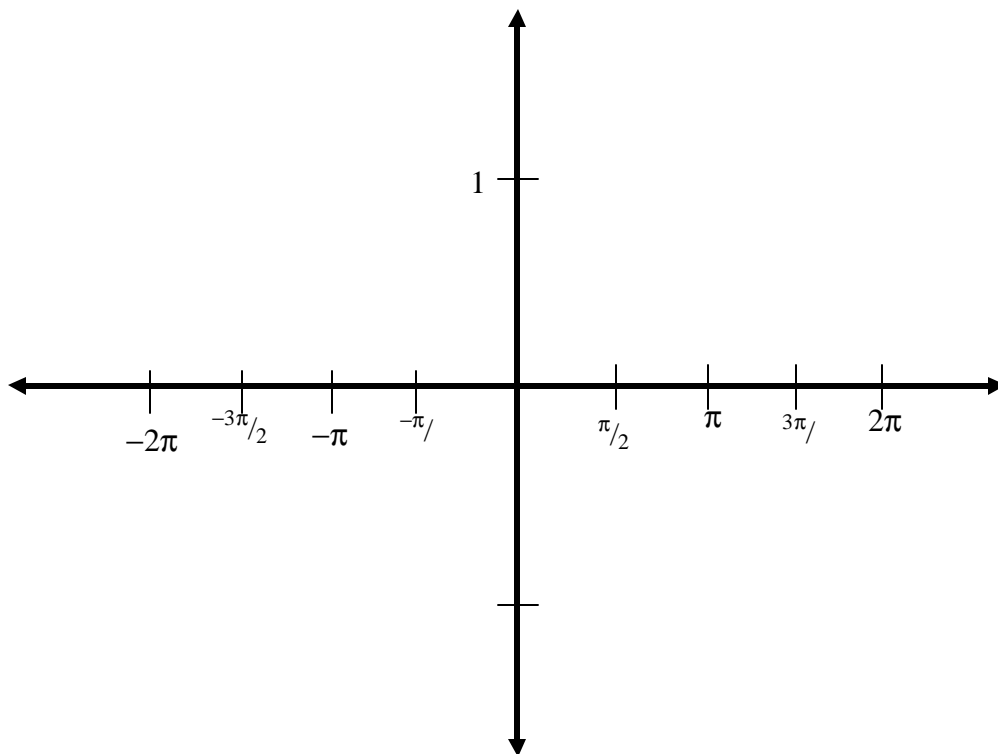
Vertical Asymptote(s):

Symmetry:

Period:

x-intercepts:

y-intercept:



**Cosecant Function:**  $f(x) = \csc x$

$x$	$\sin x$	$\csc x$
$0$		
$\frac{p}{2}$		
$p$		
$\frac{3p}{2}$		
$2p$		

Domain:

Range:

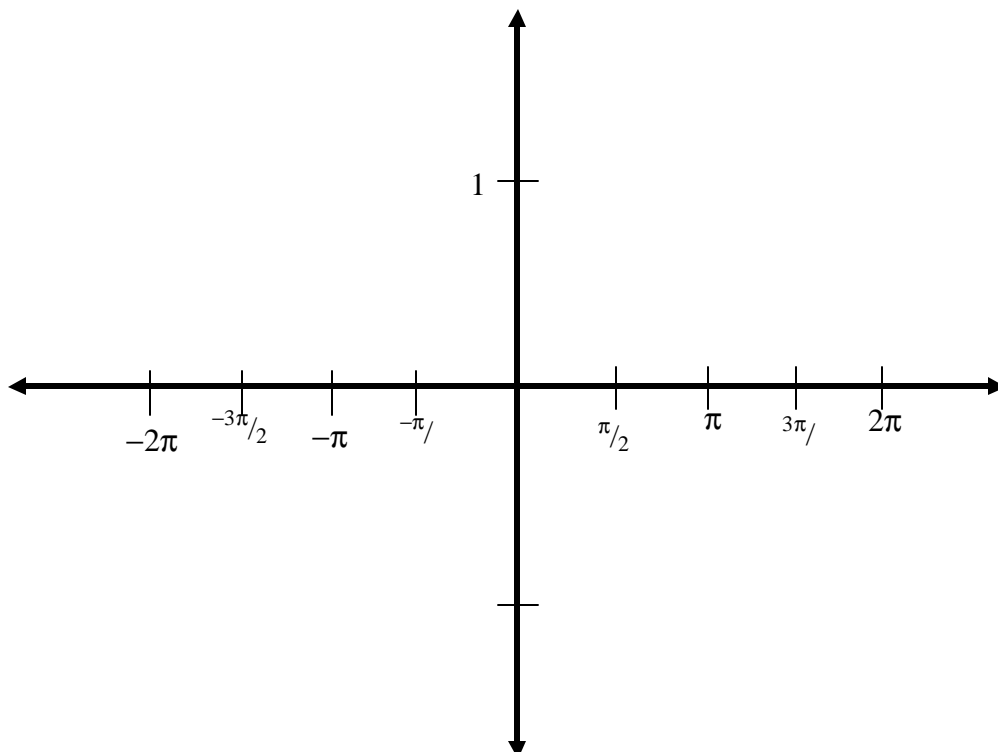
Vertical Asymptote(s):

Symmetry:

Period:

x-intercepts:

y-intercept:



**Sinusoidal Graphs**  $f(x) = A\sin(\omega x - \mathbf{f}) + B$  and  $f(x) = A\cos(\omega x - \mathbf{f}) + B$

$$\sin x = \cos\left(x - \frac{\mathbf{p}}{2}\right)$$

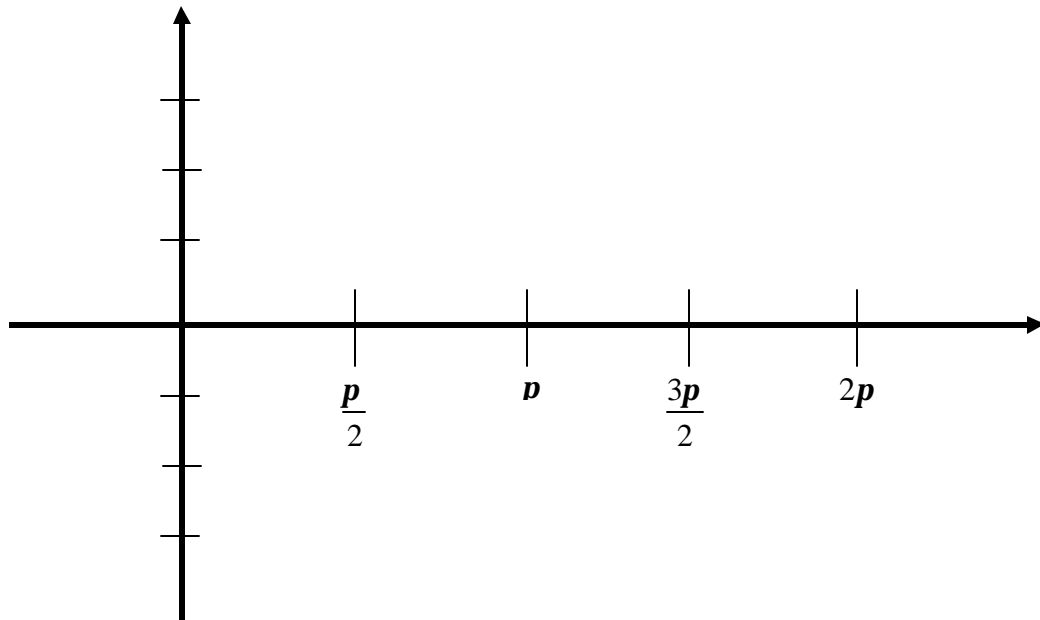
**Amplitude:**  $f(x) = A\sin x$  and  $f(x) = A\cos x$

Graph the following on the axes.

$$f(x) = \sin x$$

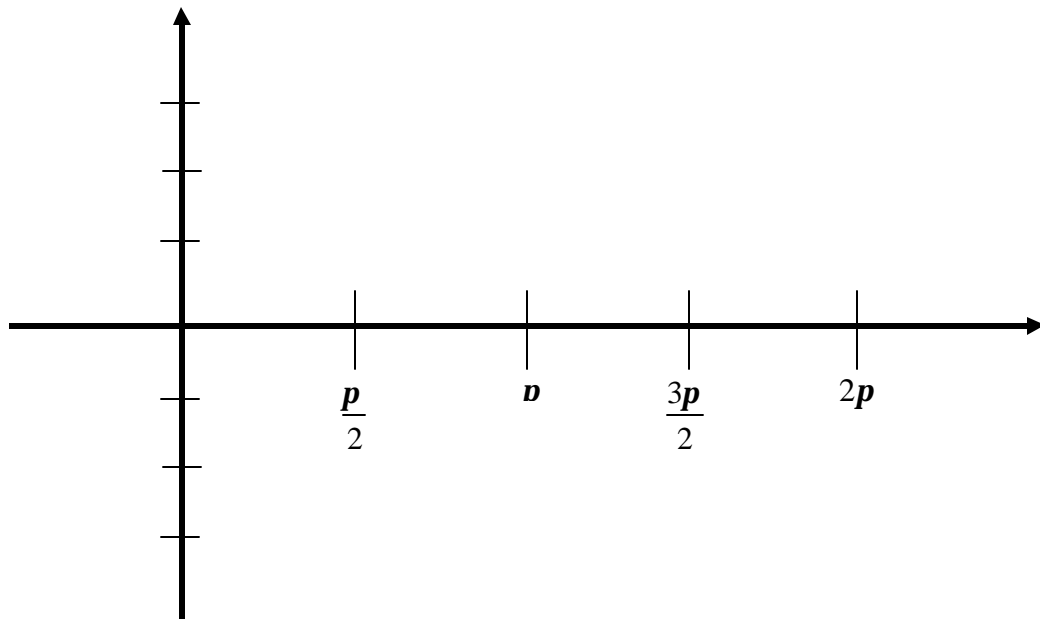
$$g(x) = 2\sin x$$

$$h(x) = 3\sin x$$



$$f(x) = \cos x$$

$$g(x) = 3\cos x$$



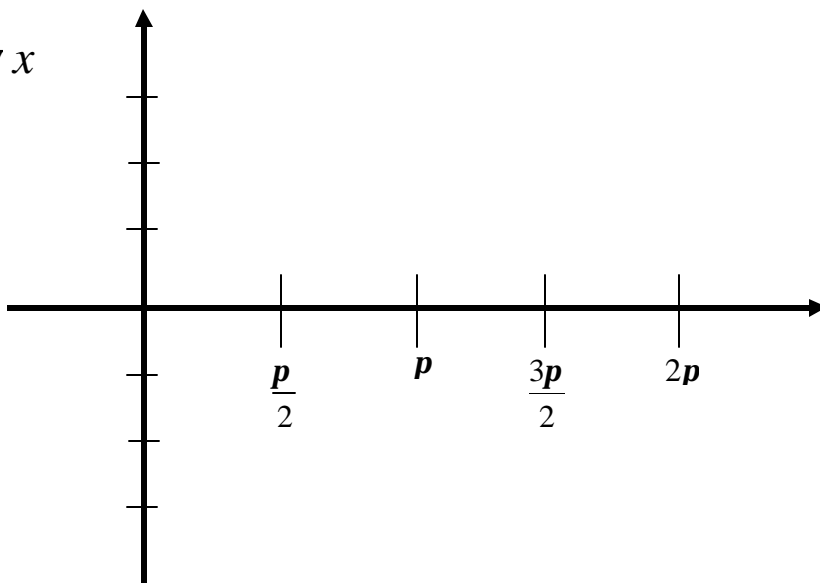
**Period:**  $y = \sin Wx$  and  $y = \cos Wx$

**Graph the following:**

$$f(x) = \sin x$$

$$g(x) = \sin 2x$$

$$h(x) = \sin \frac{x}{2}$$

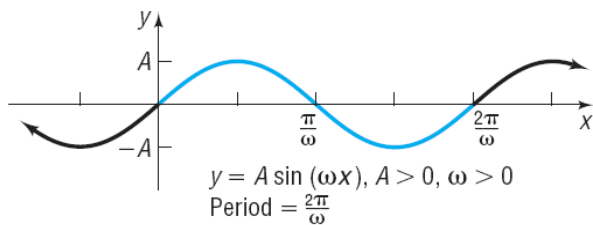
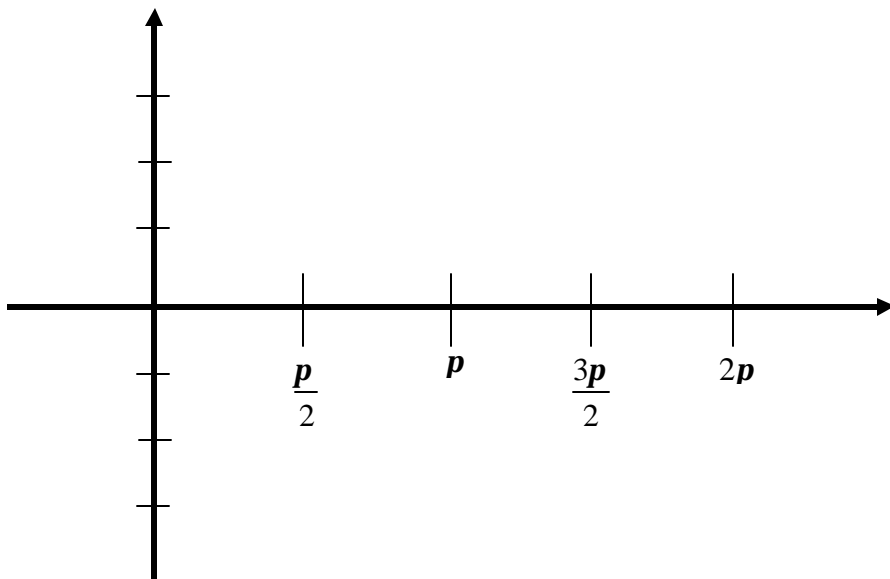


**Graph the following:**

$$f(x) = \cos x$$

$$g(x) = \cos 2x$$

$$h(x) = \cos \frac{x}{2}$$



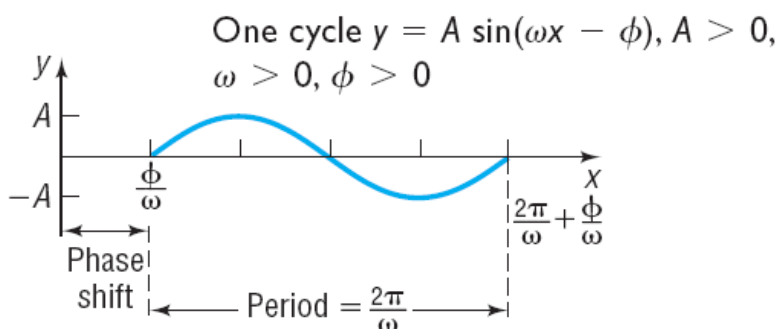
**Theorem**

If  $\omega > 0$ , the amplitude and period of  $y = A \sin(\omega x)$  and  $y = A \cos(\omega x)$  are

Amplitude = $ A $	Period = $T = \frac{2\pi}{\omega}$
-------------------	------------------------------------

**SUMMARY** Steps for Graphing a Sinusoidal Function of the Form  $y = A \sin(\omega x)$  or  $y = A \cos(\omega x)$  Using Key Points

- STEP 1:** Determine the amplitude and period of the sinusoidal function.
- STEP 2:** Divide the interval  $\left[0, \frac{2\pi}{\omega}\right]$  into four subintervals of the same length.
- STEP 3:** Use the endpoints of these subintervals to obtain five key points on the graph.
- STEP 4:** Plot the five key points with a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.



For the graphs of  $y = A \sin(\omega x - \phi)$  or  $y = A \cos(\omega x - \phi)$ ,  $\omega > 0$ ,

Amplitude =  $|A|$     Period =  $T = \frac{2\pi}{\omega}$     Phase shift =  $\frac{\phi}{\omega}$

The phase shift is to the left if  $\phi < 0$  and to the right if  $\phi > 0$ .

**SUMMARY** Steps for Graphing Sinusoidal Functions  $y = A \sin(\omega x - \phi) + B$  or  $y = A \cos(\omega x - \phi) + B$

- STEP 1:** Determine the amplitude  $|A|$  and period  $T = \frac{2\pi}{\omega}$ .
- STEP 2:** Determine the starting point of one cycle of the graph,  $\frac{\phi}{\omega}$ . Determine the ending point of one cycle of the graph,  $\frac{\phi}{\omega} + \frac{2\pi}{\omega}$ . Divide the interval  $\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + \frac{2\pi}{\omega}\right]$  into four subintervals, each of length  $\frac{2\pi}{\omega} \div 4$ .
- STEP 3:** Use the endpoints of the subintervals to find the five key points on the graph.
- STEP 4:** Plot the five key points with a sinusoidal graph to obtain one cycle of the graph. Extend the graph in each direction to make it complete.
- STEP 5:** If  $B \neq 0$ , apply a vertical shift.

**Phase Shift:**  $y = \sin(\mathbf{w}x - \mathbf{f})$  and  $y = \cos(\mathbf{w}x - \mathbf{f})$ ,  $w > 0$

Sketch one cycle of the graph of  $y = \sin(2x - p) =$  \_\_\_\_\_

**Steps for Fitting Data to a Sine Function  $y = A \sin(\omega x - \phi) + B$**

**STEP 1:** Determine  $A$ , the amplitude of the function.

$$\text{Amplitude} = \frac{\text{largest data value} - \text{smallest data value}}{2}$$

**STEP 2:** Determine  $B$ , the vertical shift of the function.

$$\text{Vertical shift} = \frac{\text{largest data value} + \text{smallest data value}}{2}$$

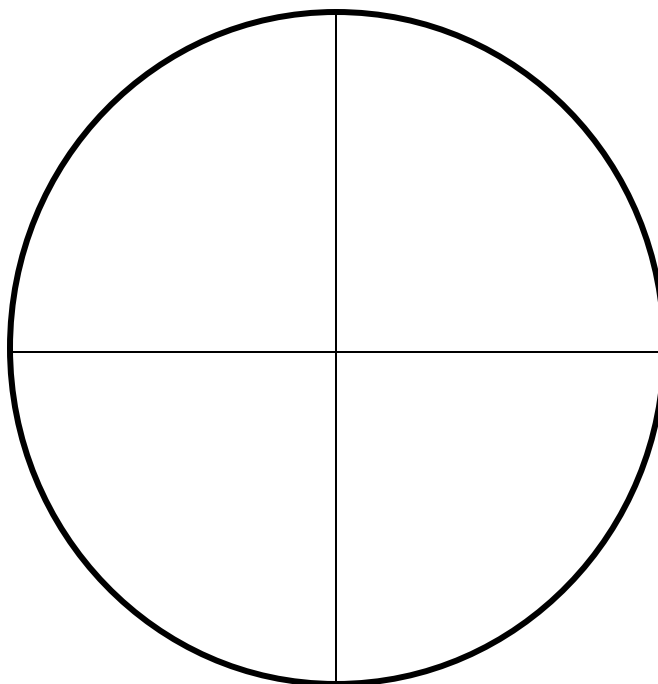
**STEP 3:** Determine  $\omega$ . Since the period  $T$ , the time it takes for the data to repeat, is  $T = \frac{2\pi}{\omega}$ , we have

$$\omega = \frac{2\pi}{T}$$

**STEP 4:** Determine the horizontal shift of the function by using the period of the data. Divide the period into four subintervals of equal length. Determine the  $x$ -coordinate for the maximum of the sine function and the  $x$ -coordinate for the maximum value of the data. Use this information to determine the value of the phase shift,  $\frac{\phi}{\omega}$ .

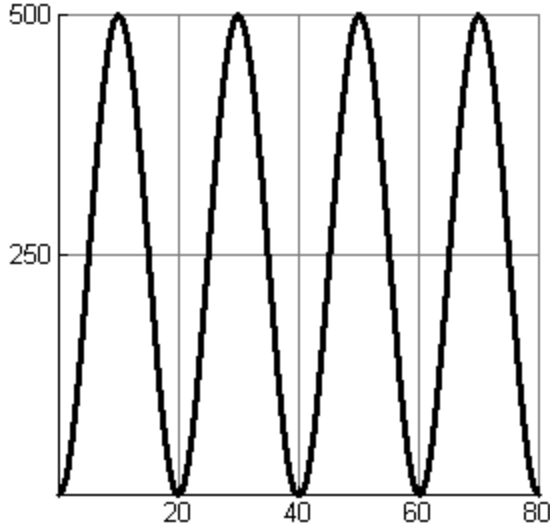
To celebrate the new millennium, British Airways announced its plans to fund construction of the world's largest ferris wheel to be located on the south bank of the river Thames in London. The ferris wheel will measure 500 feet in diameter, will carry up to 1400 passengers in 60 capsules, and will turn continuously completing a single rotation once every 20 minutes.

$t$ (min)	$f(t)$ (feet)
5	
10	
15	
20	
25	
30	
35	
40	
45	
50	
55	
60	
65	
70	
75	
80	

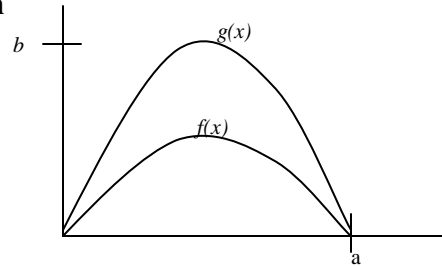


Sinusoidal Functions

1. Find two functions  $f(x) = A \cos(\omega x) + B$  and  $f(x) = A \sin(\omega x - \phi) + B$  for the graph below.



2. The graph shows the graphs of  $y = \sin x$  and  $y = 2 \sin x$ . Which function is  $y = 2 \sin x$ ? Find values of  $a$  and  $b$ .



3. Household electrical power in the US is provided in the form of alternating current. Typically voltage cycles smoothly between +155.6 volts and -155.6 volts 60 times per second. Use a cosine function to model the alternating voltage.

4. Describe the shifts of the graph of  $g$  with respect to the graph of  $f$ :

$$g(x) = -3 + \sin\left(4x + \frac{\pi}{2}\right) \text{ and } f(x) = \sin(4x)$$

- |                                 |                                  |
|---------------------------------|----------------------------------|
| a) $\frac{\pi}{8}$ left, 3 down | b) $\frac{\pi}{8}$ right, 3 down |
| c) $\frac{\pi}{2}$ left, 3 down | d) $\frac{\pi}{2}$ right, 3 up   |