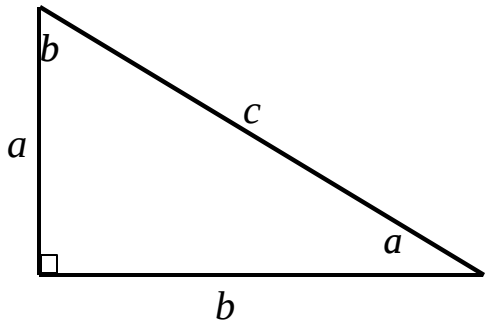
Scott
Precalculus MA 180
Solving Triangles (8.1 - 8.3)

a)	Draw a triangle that gives a visual representation of the problem. Show known quantities on the triangle and use variables to indicate unknown quantities.
b)	Write an equation and solve.



To solve a right triangle means to find the missing lengths of its sides and the measurements of its angles.

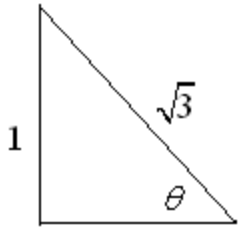
Solve right triangles using the Pythagorean Theorem and right triangle trigonometry.

1. In a right triangle, where C is the right angle, $a = 111.4\ cm$ and $b = 125.3\ cm$. Find c, A, and B.

2. A 500-foot cliff drops vertically into the ocean. If the angle of depression from the top of the cliff to a ship is 42°, how far off shore, to the nearest foot, is the ship?

3. A security camera in a neighborhood bank is mounted on a wall 10 feet above the floor. What angle of depression should be used if the camera is to be directed to a spot 7 feet above the floor and 13 feet from the wall?

4. Find the six trigonometric functions of the angle q in the figure below.



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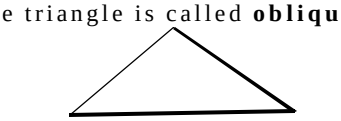
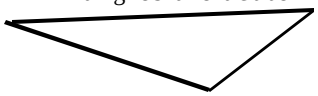
If none of the angles of a triangle is a right angle, the triangle is called **oblique.**



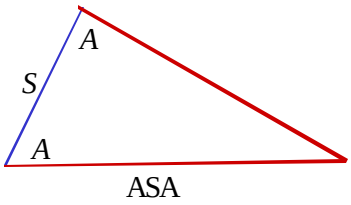
All angles are acute



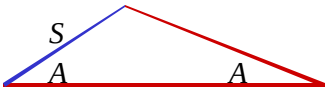
Two acute angles, one obtuse angle

To solve an **oblique triangle** means to find the lengths of its sides and the measurements of its angles.

Law of Sines: Used to solve oblique triangles. Cases: ASA, SAA, Ambiguous Case - SSA

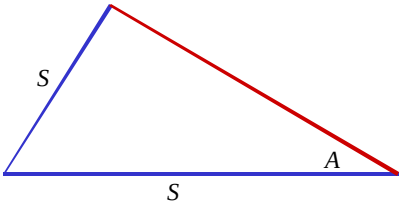


ASA



SAA

CASE 1: ASA or SAA



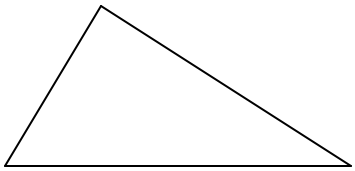
CASE 2: SSA

Ambiguous Case

Theorem Law of Sines

For a triangle with sides a, b, c and opposite angles a, b, g, respectively,

$$\frac{\sin a}{a} = \frac{\sin b}{b} = \frac{\sin g}{c}$$



$$a + b + g = 180^\circ$$

1. $A = 37^\circ,\ C = 75^\circ,\ a = 11$

2. $a = 44,\ b = 62,\ A = 42^\circ$

3. $b = 15,\ c = 25,\ B = 63^\circ$

4. Two fire-lookout stations are 30 miles apart, with Station B directly west of Station A. Both stations spot a fire on a mountain to the south. The bearing from station A to the fire is $N32^\circ W$. The bearing from station B to the fire is $N40^\circ E$. How far, to the nearest mile, is the fire from station B?

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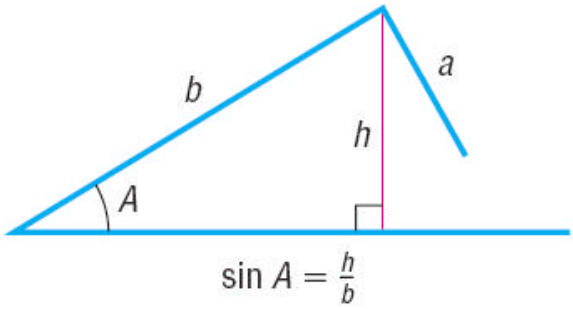
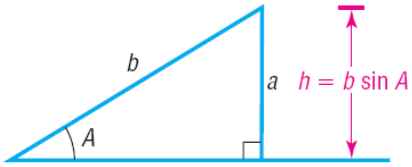
SSA --- The Ambiguous Case



One Right Triangle If $a = h = b \sin A$, then side a is just long enough to form a right triangle. See Figure 15.

Figure 15
$a = h = b \sin A$

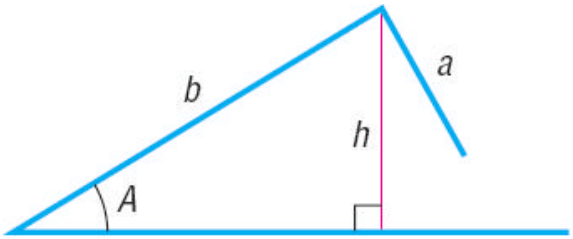


One Triangle If $a \geq b$, only one triangle can be formed. See Figure 17.

Figure 17
$a \geq b$

a)	Draw a triangle that gives a visual representation of the problem. Show known quantities on the triangle and use variables to indicate unknown quantities.
b)	Write an equation and solve.

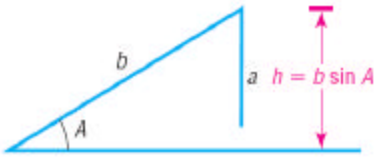
SSA --- The Ambiguous Case



$$\sin A = \frac{h}{b}$$

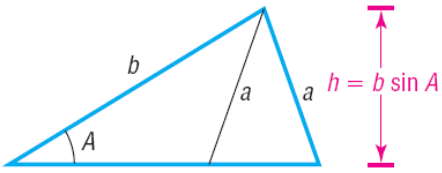
No Triangle If $a < h = b \sin A$, then side a is not sufficiently long to form a triangle. See Figure 14.

Figure 14
$a < h = b \sin A$



Two Triangles If $h = b \sin A < a$, and $a < b$ two distinct triangles can be formed from the given information. See Figure 16.

Figure 16
$b \sin A < a$ and $a < b$



a) Draw a triangle that gives a visual representation of the problem. Show known quantities on the triangle and use variables to indicate unknown quantities.
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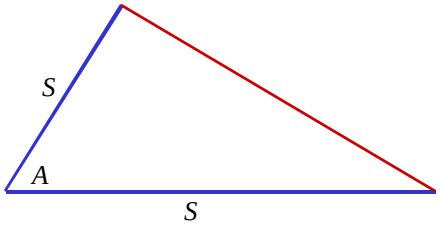
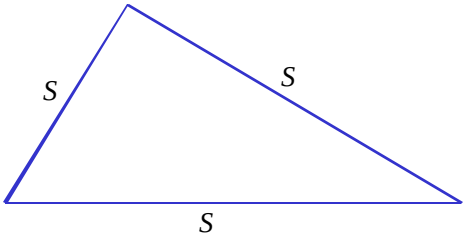
Law of Cosines: used to solve oblique triangles. Cases: SSS and SAS



CASE 3: SAS



CASE 4: SSS

Theorem Law of Cosines
For a triangle with sides a, b, c and opposite angles α, β, γ, respectively.

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Solve the oblique triangles.

1. $a = 26,\ b = 32,\ C = 110^\circ$

2. $a = 32,\ b = 46,\ c = 69$

3. $b = 6,\ c = 3,\ A = 70^\circ$

4. A plane leaves airport A and travels 690 miles to airport B on a bearing of N 42° E. The plane later leaves airport B and travels to airport C 525 miles away on a bearing of S 81° E. Find the distance from airport A to airport C to the nearest mile.

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Solutions:

Section 8.1

Answer: *1) c=167.7 cm,* $A = 41.64^\circ, B = 48.36^\circ$ *; 2) 555 feet; 3)* 13° *;*
4) $\sin q = \frac{\sqrt{3}}{3},\ \cos q = \frac{\sqrt{6}}{3},\ \tan q = \frac{\sqrt{2}}{2},\ \cot q = \sqrt{2},\ \csc q = \sqrt{3},\ \sec q = \frac{\sqrt{6}}{2}$

Section 8.2

Answer: *1)* $B = 68^\circ,\ b = 16.9,\ c = 17.7$ *; 2)* $B_1 = 71^\circ,\ C_1 = 67^\circ,\ c_1 = 60.5,$
$B_2 = 109^\circ,\ C_2 = 29^\circ,\ c_2 = 31.9$ *; 3) No Solution; 4) 27 miles*

Section 8.3

Answer: *1)* $c = 47.6,\ A = 31^\circ,\ B = 39^\circ$ *; 2)* $A = 23^\circ,\ B = 34^\circ,\ C = 123^\circ$ *;*
3) $a = 5.7,\ C = 29.6^\circ,\ B = 80.4^\circ$ *; 4) 1071 miles*