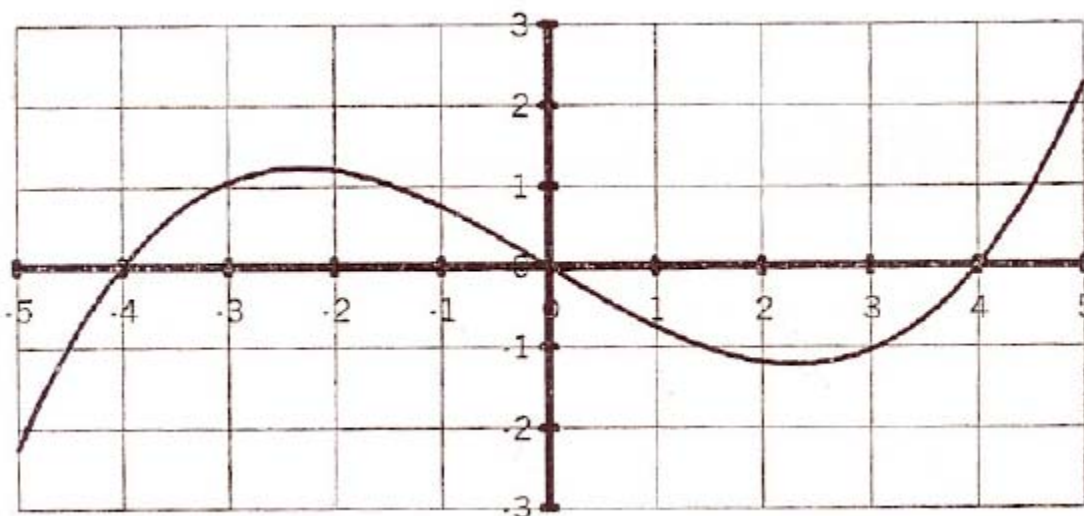


The Derivative as a Function (2.7)

Below is the graph of $f(x)$.

- For each point x in the table, draw the line tangent to x at the point $(x, f(x))$, estimate the slope of the tangent line, and fill in the table
- Graph the function $f'(x)$



| | | | | | | | | | | | |
|---------|------|----|----|------|----|---|---|---|-----|---|---|
| x | -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| $f(x)$ | -2.3 | | | 1.2 | | | | | -1 | | |
| $f'(x)$ | 3 | | | -0.2 | | | | | 0.5 | | |

http://mathdemos.gcsu.edu/mathdemos/function_sketch/sketch_the_function.html

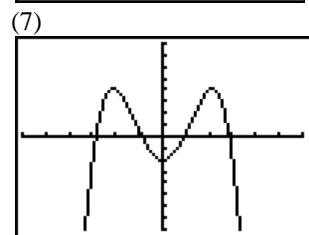
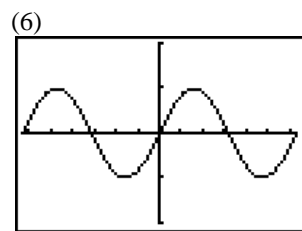
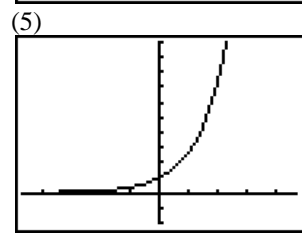
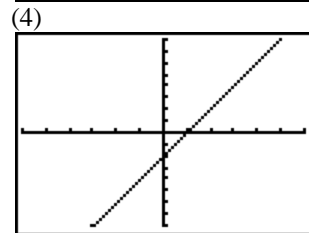
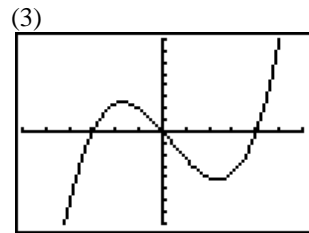
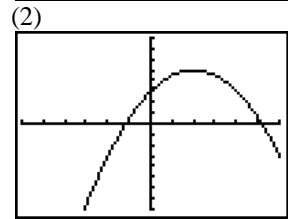
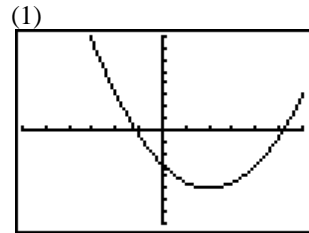
http://mathdemos.gcsu.edu/mathdemos/derivative_sketch/sketch_the_derivative_gallery.html

- The function f' is called the derivative of f because it has been “derived” from f .
- The domain of f' is the set $\{x \mid f'(x) \text{ exists}\}$ and may be a subset of the domain of f .

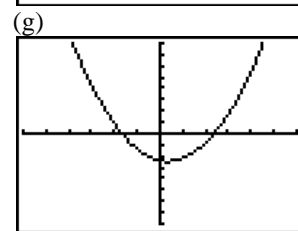
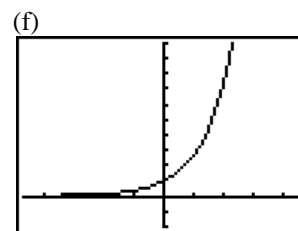
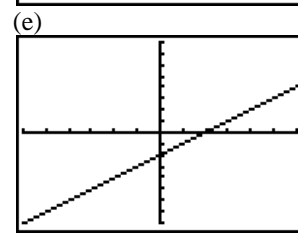
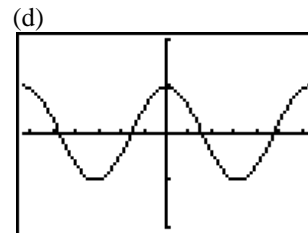
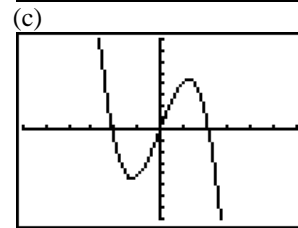
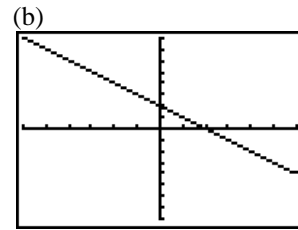
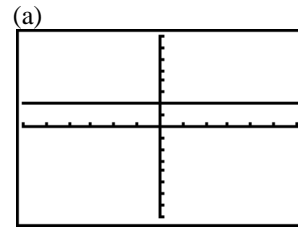
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Match the derivatives with the functions.

FUNCTIONS



DERIVATIVES



Continuity and Differentiability (2.8)

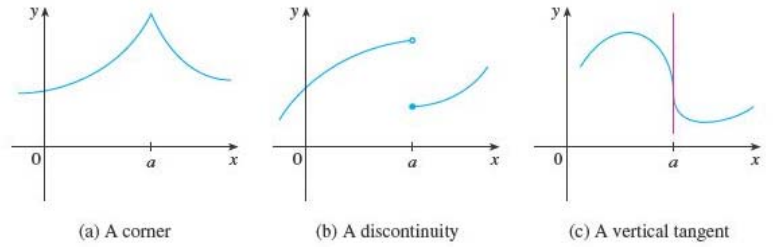
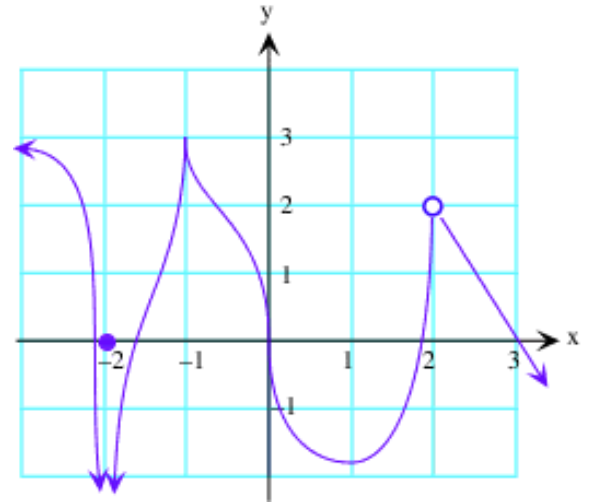


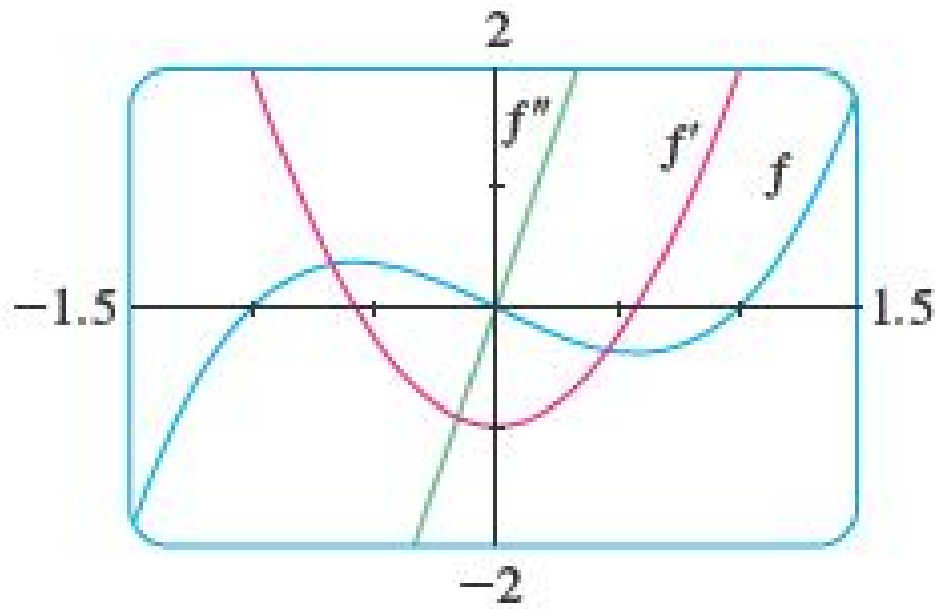
FIGURE 8
 Three ways for f not to be differentiable at a

The graph of a function $f(x)$ is sketched at the right. Use the graph to complete the chart.

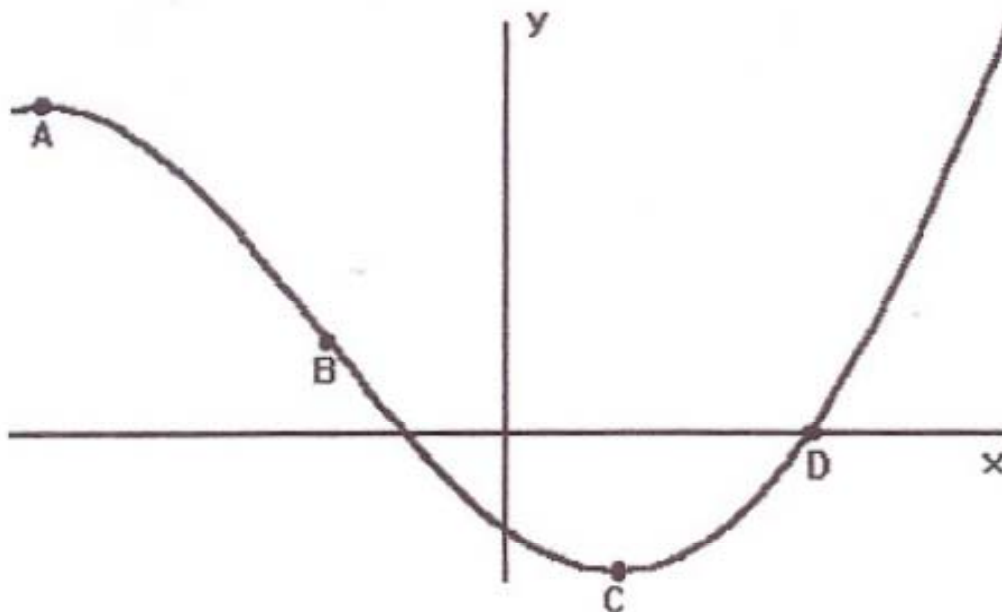


| | $a = -2.5$ | $a = -2$ | $a = -1$ | $a = 0$ | $a = 1$ | $a = 2$ | $a = 3$ |
|------------------------------------|------------|----------|----------|---------|---------|---------|---------|
| $f(a)$ | | | | | | | |
| $\lim_{x \rightarrow a} f(x)$ | | | | | | | |
| $f'(a)$ | | | | | | | |
| Is f continuous at $x = a$? | | | | | | | |
| Is f differentiable at $x = a$? | | | | | | | |

Higher Derivatives

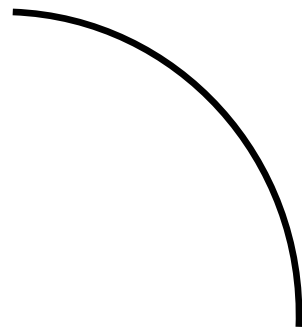
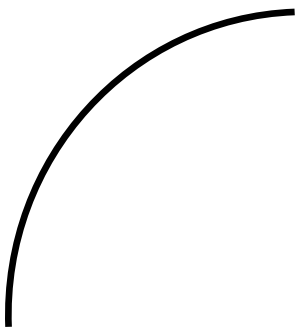
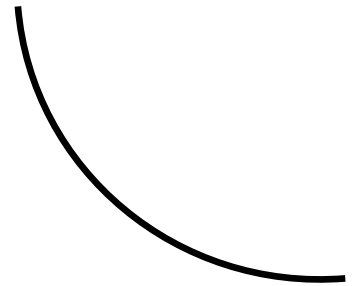
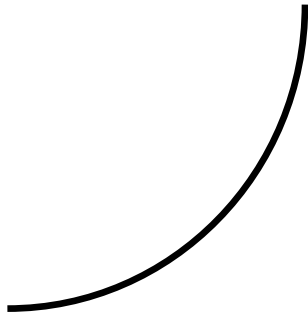


The following is the graph of the function f .



1. At the point A the function f is _____ (positive, negative zero), the first derivative f' is _____ (positive, negative zero), and the second derivative f'' is _____ (positive, negative zero).
2. At the point B the function f is _____ (positive, negative zero), the first derivative f' is _____ (positive, negative zero), and the second derivative f'' is _____ (positive, negative zero).
3. At the point C the function f is _____ (positive, negative zero), the first derivative f' is _____ (positive, negative zero), and the second derivative f'' is _____ (positive, negative zero).
4. At the point D the function f is _____ (positive, negative zero), the first derivative f' is _____ (positive, negative zero), and the second derivative f'' is _____ (positive, negative zero).

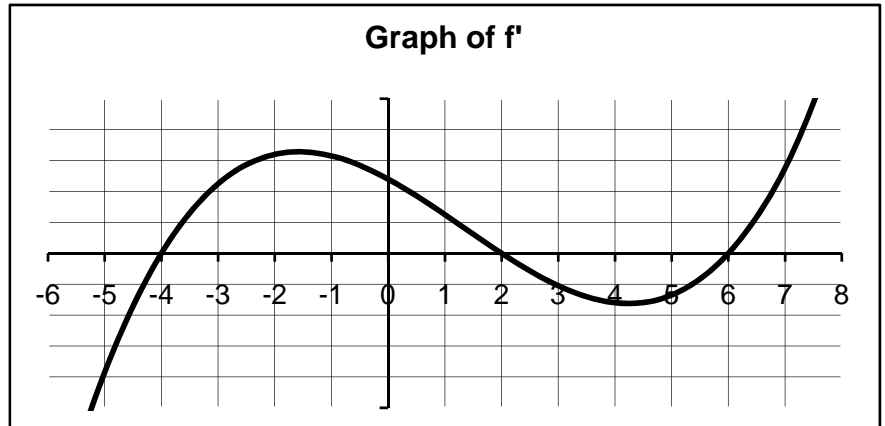
What does f say about f' ? f'' ?



Using the Graph of the Derivative (2.8)

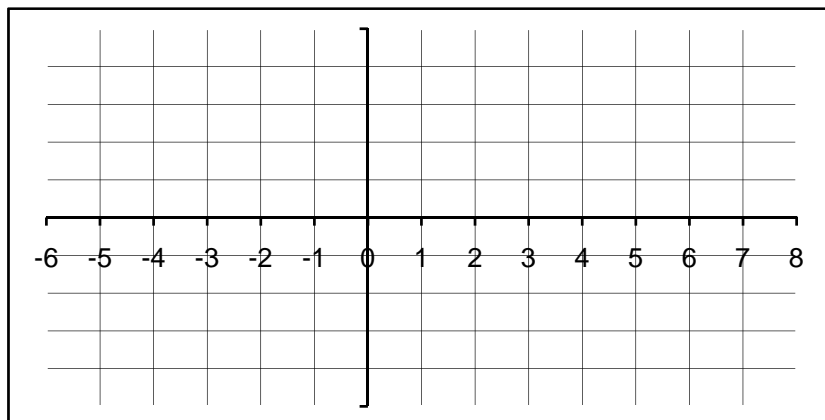
The graph to the right is the graph of f' , the *derivative* of a function f . Note that the graph of f is not shown.

If the function f is defined for all x , use this graph to answer the following questions.



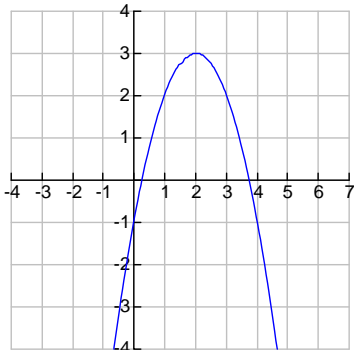
1. On what interval(s) is the function f increasing?
2. On what interval(s) is the function f decreasing?
3. At what value(s) of x , if any, does f have a local maximum?
4. At what value(s) of x , if any, does f have a local minimum?
5. On what interval(s) is the function f concave upward?
6. On what interval(s) is the function f concave downward?
7. At what value(s) of x does f have an inflection point?

Suppose it is also known that f goes through the point $(0,0)$. Based on all of the above information, sketch a possible graph of the function f .

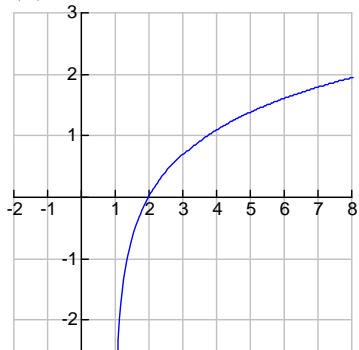


First and Second Derivatives (2.8)

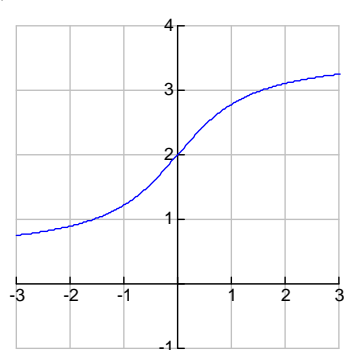
(a)



(b)



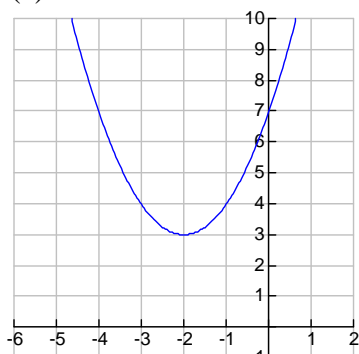
(c)



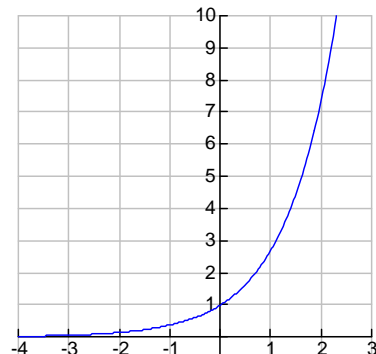
(d)



(e)



(f)



Refer to the functions whose graphs are shown above to answer the following questions.

1. Which functions have a *positive first derivative* for all x ? Explain.

2. Which functions have a *negative first derivative* for all x ? Explain.

3. Which functions have a *positive second derivative* for all x ? Explain.

4. Which functions have a *negative second derivative* for all x ? Explain.

5. Determine which function is the derivative of the other.

